The Social Value of Public Information When Not Everyone is Privately Informed

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Abstract

When there is strategic complementarity and all agents have access to public information, but only a subset of them has access to private information, strategic complementarity within the subset of privately-informed agents enhances the focal power of public information. The resulting expected social welfare function is always convex in the precision of both private and public information, compared to the symmetric information case. The welfare gain from increasing the precision of the public information always exceeds the welfare loss from the underutilization of private information by a subset of agents. The findings are robust to several extensions such as biased perceptions about public signals and costly acquisition of private information. The results support the use of public information campaigns to change agent behavior regarding vaccine hesitancy and social injustices, and may also shed light on why consumer expectations of economic variables consistently differ from professional forecasts.

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1 Introduction

In situations where agents have incentives to coordinate their actions, public information serves as a focal point in addition to providing information that is useful to individual decision-making. However, as Morris and Shin [2002] pointed out, the presence of a coordination motive forces agents to calculate higher-order beliefs on the average action. The commonality of public information across all agents ensures that it does not "wash out" when calculating average actions, leading agents to underweigh even very precise private information. If all agents have private information, this results to a social welfare loss relative to the case where there is no coordination motive. Moreover, social welfare has been shown to decrease when the ratio of precisions of public information to private information exceeds a threshold.

These results have found wide use in general equilibrium, central banking, and asset pricing settings, where all agents are assumed to have imperfect private and public information regarding the true value of the relevant state.[1] Angeletos and Pavan [2007] characterized these settings as having excessively high equilibrium levels of coordination among agents relative to what is socially optimal. The main implication across this set of papers is that as agents overweigh the public signal, prices become more and more sluggish and uninformative as time goes by. As such, the policy recommendation has been to ensure that public information should not be very precise, in order to reduce the inefficient underutilization of private information.

However, there still exist many situations where actions are strategic complements, yet not all agents have access to the same type and quality of information. This is different from the other cases considered by Angeletos and Pavan [2007] in that the agents here are not symmetric in their information structure. The continued use of public information campaigns as a policy instrument to influence behavior of agents suggests that social planners believe that some agents do not have access to relevant information on issues that are deemed important, and where agents value a certain degree of conformity. One may argue that access to the internet should alleviate this problem, but even individuals who have access to the internet face barriers such as paywalls and subscription models.[2] Moreover, it remains a fact that acquisition of private information is prohibitively high for individuals who live in nations with underdeveloped infrastructure. While the number of individuals who have internet access has more than doubled from 2 billion in 2010 to 4.1 billion in 2019, there is a large disparity among nations – only 19.1 percent of individuals from least developed countries have access to the internet, compared to 86.6 percent of individuals from developed countries [ITU and UNESCO, 2020]. This means that many individuals would not be able to obtain private information simply because they

[1] Papers in the general equilibrium tradition include Morris and Shin [2006], Amato and Shin [2006], Nimark [2008], and Angeletos and La'O [2009], Angeletos et al. [2016] provide an excellent discussion of papers that use strategic complementarity of agents actions as an information friction and how they differ from other approaches. In financial markets, Allen et al. [2006] is a notable example. Walsh [2007] is an example dealing with the effects of central bank announcements on monetary policy.

[2] Newman et al. [2021] report that only 17 percent of surveyed individuals among 20 countries in 2020 report paying for online news subscriptions.

do not have the means to do so. Knowledge would only come to them from public information campaigns.

To accommodate these issues, I modify the Morris and Shin [2002] model such that only a subset of agents has access to private information, while maintaining uniform access to public information. While this assumption can be motivated by the examples above, the measure of agents can be endogenously derived by embedding the model in a two-stage game where the agents are free to choose whether or not they would like to obtain private information at a cost. To the best of my knowledge, this information structure has remained unmodeled in the literature.

In this setting, there turns out to be strategic complementarities in the actions chosen by privately-informed agents - their actions are increasing in the measure of agents who have private information. At the same time, their actions are increasing in the overall strategic complementarity parameter. The interaction of these two measures reduces the weight on private signals by the privately-informed agents, which enhances the focal power of public information relative to the Morris and Shin [2002] case. As such, expected social welfare is always increasing in the precision of both private and public information. The enhanced role of public information ensures that it is always beneficial to increase its precision. The reason is that the inefficiency from underusing private information is restricted to only the privately-informed agents, so that the welfare gain from increasing precision of public information more than offsets the welfare loss from underusing private information.

The extension of the Morris and Shin [2002] model allows application not only to purely economic settings with informationally-rich agents, but also to wider contexts such as mitigating public health crises involving contagious diseases, as well as correcting social injustices such as misogyny, racism, and stereotyping.

This paper is related to the family of models that reduce overreaction to public information, following Morris and Shin [2002]. Papers that are directly related to the methodology include Cornand and Heinemann [2008] and Baeriswyl and Cornand [2014]. These papers restrict public information to only a subset of agents, while I restrict private information to only a subset of agents. As such, my setting enhances the power of public information and highlights the strategic complementarity that arises in the privately-informed subset of agents. In contrast, no such thing arises in the reverse cases of Cornand and Heinemann [2008] and Baeriswyl and Cornand [2014], as the measure of publicly-informed agents in their case is a direct reduction to the overall strategic complementarity parameter.

This paper is also related to the literature on costly information acquisition within the beauty contest framework, as I allow the measure of agents that acquire private information to be endogenously determined. The closest paper to this end is Arato et al. [2021], who find that imposing nonlinear pricing on public information eliminates the multiple equilibria problem found by Hellwig and Veldkamp [2009]. More generally, Hellwig and Veldkamp [2009] show in games with strategic complementarities in actions that agents tend to acquire the information that other agents already possess in order to enhance coordination. While Hellwig and Veldkamp [2009] state that a unique equilibrium is guaranteed when the information acquired is private, I find that it is no longer true when not all agents have access to private information.

Other related papers in the costly information acquisition literature deal with the precision of information as a choice variable. Colombo and Femminis [2008] find that precise public information by a social planner crowds out the agents' acquisition of more precise private information. Colombo et al. [2014] find that with endogenous acquisition of private information, the precision of costly private information desired by individuals is different from what the social planners would like them to acquire in the sense of Angeletos and Pavan [2007]. Other studies have gone beyond the traditional public-private dichotomy. Hellwig and Veldkamp [2009] identify "potentially public information" as public in the Morris and Shin [2002] sense only to the extent of the intersection of the information sets of all the agents in the model. Myatt and Wallace [2012] endogenize the level of publicity by allowing agents to choose the amount of attention paid to information. The more agents that pay attention, the higher the resulting correlation of information, thereby increasing its publicity.[3]

As to how individuals actually go about acquiring their information, newspapers have given way to online news and social media sites [Pew Research Center, 2021a]. Younger individuals tend to be more likely to obtain their news from social media despite their misgivings about the accuracy of news, while overall willingess to pay for news subscriptions remains low [Pew Research Center, 2021b, Newman et al., 2020, 2021]. As such they may be reliant on news aggregators and free websites. However, Cagé et al. [2020] documented that for the French case, copied news contains only 9.4 percent of the original source, which may be interpreted as a nonnegligible drop in precision. Moreover, news aggregators have been taken down in several jurisdictions and doing so results to a drop in news consumption by individuals [Athey et al., 2021, Chiou and Tucker, 2017]. Finally, there are issues regarding the circulation of fake news, or biased information due to few regulations imposed on social media content.[4] Fake news covers topics ranging from health to politics. Allcott and Gentzkow [2017] found a sharp increase in fake news in the run-up to the 2016 US elections. Allcott et al. [2019] find that the spread of misinformation through Facebook had been rising from 2015 to 2016 and dropped after, but the spread of misinformation through Twitter had been rising from 2015 to 2018. Fake news has also been linked to vaccine hesitancy and lowered child immunization rates for all types of vaccines in Italy as shown by Carrieri et al. [2019]. More recently, belief in conspiracy theories regarding the COVID-19 vaccine have been associated with a reduction in preventive behavior such

[3]This is related to the rational attention literature as started by Sims [2003, 2006] and found in applications such as Maćkowiak and Wiederholt [2009, 2012], Matějka and McKay [2012, 2015] and Reis [2006a, 2006b]. Here I restrict the detailed review to papers in the Morris and Shin [2002] tradition.

[4]Gentzkow et al. [2015] provide a model that rationalizes the emergence of fake news outlets because consumers' utility is increasing in receiving information that is consistent with their biases.

as mask-wearing among US adults [Romer and Jamieson, 2020]. Clearly, the prevalence of misinformation is detrimental to social welfare.

The paper proceeds as follows: Section 2 presents the model setup and main results while Section 3 presents extensions. Section 4 contains a discussion of related issues and applications while Section 5 concludes. Section 6 contains the Appendix.

2 Baseline model and main results

In this section, I introduce my baseline model. All agents have access to the same public information, but only a measure $q < 1$ of them also have access to private information.[5] That is, a measure q agents obtain public and private signals, while the rest only obtain the public one. In this section, I take the measure of agents who have access to private information (q) as a given, and show in Section 3 how this measure can be endogenized.

2.1 Setup

2.1.1 Perfect information and common knowledge about the state θ

There is a continuum of agents indexed with i, where $i \in [0, 1]$. The agents have a utility function as in Morris and Shin [2002]:

$$u_i(\boldsymbol{a}, \theta) = -\left[(1-r)(a_i - \theta)^2 + r\left(L_i - \bar{L}\right)\right] \tag{1}$$

where

$$L_i = \int_0^1 (a_j - a_i)^2 \, dj \tag{2}$$

and

$$\bar{L} = \int_0^1 L_j dj. \tag{3}$$

θ denotes the true economic state, while $\boldsymbol{a}$ is the vector of actions taken by the agents in the continuum. The utility of agent i depends on the deviation of their own action a_i from θ, and the deviation of their own action a_i from the actions a_j of their opponents. r is the weight of the coordination element, while $1-r$ is the weight of the closeness of action a_i to the true state θ. The higher r is, the greater the coordination motive of the agents. I restrict r to be in $(0, 1)$ as I only consider cases where actions are strategic complements.

[5] This structure is the reverse of those found in Cornand and Heinemann [2008] and Baeriswyl and Cornand [2014], where all agents have access to private information, but only a measure $p \leq 1$ have access to public information. The setup where $p \leq 1$ is termed partial publicity by Cornand and Heinemann [2008] and Baeriswyl and Cornand [2014], and it is a means of suppressing the power of public information on the coordination of agents.

The agent's optimal choice of a_i can be found by maximizing (1) with respect to a_i, yielding the first-order condition

$$a_i = (1-r)\theta + r\bar{a}, \tag{4}$$

where $\bar{a} = \int_0^1 a_j dj$ for all j.

When there is perfect information about θ that is common knowledge, the first-order condition reduces to $a_i = \theta$. As in Morris and Shin [2002], define social welfare as a normalized form of the integral of (1) over the set of all agents i:

$$W(\boldsymbol{a}, \theta) = \int_0^1 \frac{u_i(\boldsymbol{a}, \theta)}{(1-r)} di = -\int_0^1 (a_i - \theta)^2 di, \tag{5}$$

which reaches its maximum at $a_i = \theta$. In this case there is no need to second-guess the actions of others as θ is common knowledge. It can be seen from (1) and (5) that while the coordination incentive is important at the individual level, it disappears upon aggregation across agents.

2.1.2 Imperfect knowledge about the state θ

Agents do not know θ but instead, they obtain signals about it. They do know that θ has an improper prior over $\mathbb{R}$.[6] All agents obtain a public signal $y = \theta + \eta$, where $\eta \sim N\left(0, \frac{1}{\alpha}\right)$, α being the precision of the public signal. The defining feature of a public signal is that it is common knowledge to all agents in the model, so that they are able to coordinate on it. Only a measure q of agents receive a private signal $x_i = \theta + \varepsilon_i$, where $\varepsilon_i \sim N\left(0, \frac{1}{\beta}\right)$, β being the precision of the private signal. There is common knowledge about the signal structure and distribution across the types of agents. That is, all agents know that a measure q of them obtain the private signal of the form $x_i = \theta + \varepsilon_i$, even though a measure $1-q$ of them do not observe x_i. As in the literature, I assume that ε_i and ε_j are uncorrelated for $i \neq j$.

When there is imperfect information about θ, the agents are unable to use (4) in its exact form. Instead, the agents have to condition on whatever information they possess. In particular, the agents use the function

$$a_i = (1-r)\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{a}), \tag{6}$$

which is the first-order condition from maximizing expected utility

$$\mathbb{E}_i[u_i(\boldsymbol{a}, \theta)] = -\mathbb{E}_i\left[(1-r)(a_i - \theta)^2 + r\left(L_i - \bar{L}\right)\right]. \tag{7}$$

[6]That θ has an improper prior over $\mathbb{R}$ does not pose a problem for the calculation of the posterior density of θ given normal signals, as explained by Morris and Shin [2002].I could also have chosen a Gaussian prior for θ, as in Colombo and Femminis [2008].

The subscript i indicates that the the expectations on θ and $\bar{a}$ are conditioned on the information set of agent i. In this setting, the information set will be different for the measure q and $1-q$ of agents. Henceforth, $a_{i,q}$ will be used to denote the optimal action of some agent i who is both privately and publicly informed, while $a_{i,1-q}$ will be used to denote the optimal action of some agent i who only has the public signal.

2.2 Individual utility: equilibrium actions of agents

Consider the case when all the agents receive the public signal y, but only q-measure of agents receive the private signal x_i. To find equilibrium, I follow Cornand and Heinemann [2008] to postulate a solution form for each group of agents separately, and calculate average action $\bar{a}$ using the two groups, taking into account the measures q and $1-q$. The assumption $q<1$ is exogenous for this analysis, but I show in Section 3 that q can be endogenized by having a stage where agents choose to acquire private information at some cost.

First, consider the $(1-q)$-measure of agents who only receive the public signal y. While the $(1-q)$-measure of agents know the information structure over the set of all agents, their solution to (6) will only be a function of y since they are only able to calculate conditional expectations about functions of θ with respect to their signal y. Bayesian updating necessitates that their optimal action is of the form $a_{i,1-q}=y$.

Consider now the q-measure of agents who receive both the public signal y and the private signal x_i. In this case, the solution to (6) will be a linear function of y and x_i. That is, for any agent i in the q-measure,

$$a_{i,q}=\kappa x_i+(1-\kappa)\,y \tag{8}$$

for some $\kappa\in(0,1)$ where κ is to be determined in equilibrium.

The average action from the viewpoint of any agent can then be calculated as the weighted average of the actions of the q- and $1-q$ measures of agents:

$$\bar{a}=q\left(\int_0^1 a_{j,q}dj\right)+(1-q)\left(\int_0^1 a_{j,1-q}dj\right)=q\kappa\theta+(1-q\kappa)\,y. \tag{9}$$

It can be seen that q has the role of an aggregator of actions - it serves as a weight to calculate the average action.

While every agent comes up with the same expression for $\bar{a}$, its conditional expection differs across agents depending on their information set. Let $\mathbb{E}_i^{1-q}(\cdot)$ and $\mathbb{E}_i^{q}(\cdot)$ denote conditional expectations of the $1-q$ and q-measures of agents, respectively. In particular, agents who receive only the public signal evaluate

$\mathbb{E}_i^{1-q}(\bar{a}) = \mathbb{E}(\bar{a}|y) = y$, while those who receive both public and private signals evaluate

$$\mathbb{E}_i^q(\bar{a}) = \mathbb{E}(\bar{a}|x_i, y) = q\kappa\left(\frac{\alpha y + \beta x_i}{\alpha+\beta}\right) + (1-q\kappa)\, y, \tag{10}$$

using

$$\mathbb{E}_i^q(\theta) = \mathbb{E}(\theta|x_i, y) = \frac{\alpha y + \beta x_i}{\alpha+\beta}. \tag{11}$$

Using (10) in (6) where $a_i = a_{i,q}$ yields

$$a_{i,q} \quad = \quad \frac{\beta(1-r+rq\kappa)}{\alpha+\beta}x_i + \left[1 - \frac{\beta(1-r+rq\kappa)}{\alpha+\beta}\right]y, \tag{12}$$

By matching the coefficients of (12) to those in (8), it can be seen that $\kappa = \frac{\beta(1-r)}{\alpha+\beta(1-rq)}$ so that the equilibrium action of an agent who has both private and public information is

$$a_{i,q} = \frac{\beta(1-r)}{\alpha+\beta(1-rq)}x_i + \left(1 - \frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)y. \tag{13}$$

For completeness, the equilibrium action of an agent who only has public information is $a_{i,1-q} = y$.

Proposition 1. *Given q, a privately-informed individual's action puts higher weight on private information as its precision increases, and puts lower weight on private information as the precision of public information increases.*

The result is intuitive - it reflects standard Bayesian updating by agents on the calculation of $\mathbb{E}_i^q(\theta)$ - an individual would put more weight to the information that is relatively more precise. Increasing the precision of private information makes public information relatively less precise, and vice versa. This is true for any r and q between 0 and 1. However, there is more to it than Bayesian updating since the strategic complementarity parameter r is also in play.

Proposition 2. *There is strategic complementarity within the subset of privately-informed agents: given precisions α and β, privately-informed agents place higher weight on private information when there are more privately-informed agents.*

The higher q is, the larger the measure of agents who are able to use x_i in their decisions, and the larger the weight that is assigned to private information x_i. In short, there are strategic complementarities in how privately-informed agents use private signals x_i, and this is measured by q. This is not obvious in a model where the agents all have access to private information, such as Morris and Shin [2002], Cornand and Heinemann [2008], Baeriswyl and Cornand [2014] and Hellwig and Veldkamp [2009]. That there are strategic complementarities in how agents use private information echoes Hellwig and Veldkamp [2009, p.229] in that there are strategic complementarities in the acquisition of private and public information.[7] Henceforth, q will also be referred to as the "private strategic complementarity" parameter, while r will be referred to as the "overall strategic complementarity" parameter.

Proposition 3. *The interaction of private strategic complementarity q with overall strategic complementarity r reduces the weight on private signals x_i relative to Morris and Shin [2002].*

Proof. By Proposition 2, the weight assigned by the q-measure of agents to private signal x_i is increasing in q. Since the implicit q in Morris and Shin [2002] is 1, the result obtains for any $q \in (0, 1)$. $\square$

One can see the intuition of this result by examining the role of q in $\mathbb{E}_i^{1-q}(\bar{a})$. Even though the q-measure of agents form the posterior beliefs about θ correctly, the beauty-contest framework leads them to (also correctly) underweigh their private information in the calculation of $\bar{a}$, because they know that only q of them have access to private information. It can be seen in (9) that the weight on private signal x_i is reduced by $q \in (0, 1)$ relative to the Morris and Shin [2002] setup and so enhances the power of public information as a focal point. The measure q interacts with the overall strategic complementarity measure r such that it pulls down the relative weight of private information, thereby increasing the relative weight of public information. The result does not contradict the previous result that by itself, an increase in q increases the weight on private signal x_i.

Corollary 1. *To maintain any given level of weight on private signal x_i, q and β are substitutes, while q and α are complements.*

There is imperfect substitutability between q and β to maintain the weight on x_i at some constant level. The more people who have private information, the less precise private information has to be. This is because while precision of private information is always a factor in determining posterior beliefs about θ, the measure of people q who are privately informed also enters into the weight by how the average action $\bar{a}$ is calculated. A marginal increase in q increases the weight on x_i by $\frac{\beta^2(1-r)r}{(\alpha+\beta(1-rq))^2}$ while a marginal increase in β increases the weight on x_i by $\frac{\alpha(1-r)}{(\alpha+\beta(1-rq))^2}$. Note that the changes are not equal. On the other hand, an increase in α always increases the weight assigned to public information, which necessarily decreases the weight on private information. In particular, a marginal increase in α reduces the weight assigned to x_i by $\frac{\alpha(1-r)}{(\alpha+\beta(1-rq))^2}$. To offset the effect of a rise in α on the weight on x_i, q must rise accordingly.

[7] While the structure in this model is the reverse of what Cornand and Heinemann [2008] and Baeriswyl and Cornand [2014] have assumed, the measure p of publicly-informed individuals in their setup cannot be interpreted as a secondary strategic complementarity parameter, instead it is a parameter that reduces overall strategic complementarity.

2.3 Expected social welfare

What is optimal at the agent level does not necessarily coincide with what is optimal at the social planner level. In this section, I consider what happens to expected social welfare under the information structure I have assumed.

By virtue of the signal forms $x_i = \theta + \varepsilon_i$ and $y = \theta + \eta$, the optimal action for privately-informed agents, (13), can be written in terms of θ as

$$a_{i,q} = \theta + \frac{\beta(1-r)}{\alpha + \beta(1-rq)}\varepsilon_i + \left(1 - \frac{\beta(1-r)}{\alpha + \beta(1-rq)}\right)\eta, \tag{14}$$

while the optimal action for publicly-informed agents, $a_{i,1-q} = y$ can be written as

$$a_{i,1-q} = \theta + \eta. \tag{15}$$

Expected social welfare can be calculated as in Cornand and Heinemann [2008], taking into account the measures q and $1-q$ - that only q agents are able to use private signals x_i. Using (14) and (15), expected social welfare is

$$\mathbb{E}(W|\theta) \quad = \quad -q\left(\frac{\alpha\beta(1-r)^2 + (\alpha + \beta r(1-q))^2}{\alpha(\alpha + \beta(1-rq))^2}\right) - \left(\frac{1-q}{\alpha}\right). \tag{16}$$

Proposition 4. *Expected social welfare is increasing in q for $q \in [0,1]$, and attains its maximum at $q = 1$.*

Proof. The derivative of expected social welfare with respect to q is

$$\frac{\partial \mathbb{E}(W|\theta)}{\partial q} = \frac{\beta(\alpha+\beta)(1-r)(\beta q r(r-3) + (\alpha+\beta)(1+r))}{\alpha(\beta(1-qr)+\alpha)^3} > 0. \tag{17}$$

This expression is positive when increasing in q for $q \leq \left(\frac{\alpha+\beta}{r\beta}\right)\left(\frac{1+r}{3-r}\right) = q^*$. The expression for q^* attains its smallest value when $\alpha = 0$, in which case q^* reduces to $\frac{1}{r}\left(\frac{1+r}{3-r}\right)$. However, this expression will only be less than 1 if $r(3-r) < (1+r)$ but this turns out to be $(1-r)^2 < 0$ which is impossible for $r \in \mathbb{R}$. Therefore the maximum is constrained at $q = 1$. □

Figure 1 presents graphs of the expected social welfare function as q increases.

[Figure 1 about here.]

As such, when acquiring information is costless, $q = 1$ maximizes the expected social welfare function, which is what Morris and Shin [2002] implicitly assumed. With costly information acquisition, the endogenous

q will not necessarily be 1 and is not a choice variable of the social planner.[8] In contrast, Cornand and Heinemann [2008] set out to find the measure $p \in [0,1]$ of agents who should be publicly informed (as opposed to only having private information) in order to maximize expected social welfare. They are able to find some $p < 1$ for a sufficiently small r because p directly reduces the influence of r. The same cannot be done here for q because the presence of q actually enhances the focal power of public information.

Henceforth, I take $q < 1$. To see the effects of increasing the precision of the private and public signals on expected social welfare given q and r, derivatives can be taken as follows.

Proposition 5. *When $q \in [0,1]$, increasing the precision of the private signal β always increases welfare.*

Proof. The derivative of $\mathbb{E}(W|\theta)$ with respect to β is

$$\frac{\partial \mathbb{E}(W|\theta)}{\partial \beta} = \frac{q(1-r)(\beta r q(r-3) + (\alpha+\beta)(1+r))}{(\alpha + \beta(1-rq))^3} \tag{18}$$

which is positive only if $\beta r q(r-3) + (\alpha+\beta)(1+r) > 0 \Leftrightarrow q < \left(\frac{\alpha+\beta}{r\beta}\right)\left(\frac{1+r}{3-r}\right) = q^*$. Since $q^* > 1$ from Proposition 4, $\beta r q(r-3) + (\alpha+\beta)(1+r) > 0$ for $q \in [0,1]$, which means that expected social welfare will always increase with the precision of private information. □

Proposition 6. *When $q \in (0,1)$, increasing the precision of the public signal α always increases welfare.*

Proof. The derivative of $\mathbb{E}(W|\theta)$ with respect to α is

$$\frac{\partial \mathbb{E}(W|\theta)}{\partial \alpha} = \frac{q\left((\beta r(q-1))^2 - \alpha^2\right)}{\alpha^2(\alpha+\beta(1-rq))^2} + \frac{2q\left(\alpha\beta(1-r)^2 + (\alpha+\beta r(1-q))^2\right)}{\alpha(\alpha+\beta(1-rq))^3} + \frac{1-q}{\alpha^2}. \tag{19}$$

The limit of this expression as $\alpha \to \infty$ is zero. □

The intuition is as follows: q's role as an aggregator reduces the weight of x_i in the calculation of $\bar{a}$ as explained in Proposition 3, and again in the calculation of expected social welfare. This means that in the aggregate, the power of public signals is enhanced even more compared to the Morris and Shin [2002] setup. It is true that the q-measure of informed agents inefficiently underuse their (potentially very precise) private signals relative to the no-coordination case. But the gain from increasing the precision of public information more than offsets any welfare loss from the underuse of private information by the q-measure of agents, because public information becomes much more effective, in addition to having total reach. That only a q-measure of agents have private information means that the rest of the agents are shielded from being inefficient. As a result, any increase in α can only increase welfare.

[8] Unlike in Cornand and Heinemann [2008] and Baeriswyl and Cornand [2014] where the measure p of agents who have access to public information is within the social planner's control, the same cannot be said for measure q.

In contrast, Morris and Shin [2002] find that at $q = 1$, every agent underuses private information, so that increasing the precision of public information will only increase social welfare if $\frac{\alpha}{\beta} < (1-r)(2r-1)$. This is depicted by upward-sloping social welfare contours in the (α, β) space whenever $\frac{\alpha}{\beta} < (2r-1)(1-r)$. However, when $q = 1$, the effect of q is subsumed into r. That is, it is no longer obvious that agents who are privately informed have strategic complementarities within their subgroup, and that only the overall strategic complementarity r remains. It turns out that the social welfare contours are only of the shape found by Morris and Shin [2002] whenever $q = 1$. For $q \in (0, 1)$, the social welfare contours resemble indifference curves in (α, β) space. This shows that expected social welfare is always increasing in both α and β. Figure 2 depicts social welfare contours in (α, β) space for various $q \in (0, 1]$.

[Figure 2 about here.]

Svensson [2006] remarked that Morris and Shin [2002]'s condition $\frac{\alpha}{\beta} > (2r-1)(1-r)$ is unlikely to be violated under reasonable assumptions and thus, social welfare is actually increasing whenever public information precision increases. However, this is done in a setting where agents are both privately and publicly informed. My paper differs in that I assume only $q < 1$ agents have access to private information.

Proposition 7. *Given α and β, $\mathbb{E}(W|\theta) \to -\frac{1}{\alpha}$ as $q \to 0$. Thus when q is small, the impact of β on social welfare disappears with q while the impact of α on social welfare remains.*

Proof. The expected social welfare function (16) can be written as

$$\mathbb{E}(W|\theta) = -q\left(\frac{\alpha\beta(1-r)^2 + (\alpha + \beta r(1-q))^2}{\alpha(\alpha + \beta(1-rq))^2} + \frac{1}{\alpha}\right) - \frac{1}{\alpha}, \tag{20}$$

where it can be seen that as $q \to 0$, only the term $-\frac{1}{\alpha}$ remains. This means that any effect of β will disappear along with q, but the effect of α is sustained. □

In my model, increasing the precision of private information is still useful because it counteracts a small q in the weight assigned by the q-measure of agents to x_i in their equilibrium actions $a_{i,q}$. As in Corollary 1, to maintain any given level of weight on private signal x_i, q and β are substitutes. And as in Proposition 1, the weight of x_i is still increasing in β. The larger β becomes, the larger the weight of x_i that is carried through to the aggregate action $\bar{a}$ for any q. However, the effect of β is limited by q, and even at $q = 1$, the effect of β can never be fully as strong as the effect of α unless $r = 0$. On the other hand, increasing the precision of public information is useful because public information is already a focal point, and its role is enhanced upon restricting the measure of agents who have access to private information. The relative weight of public information in the aggregate action $\bar{a}$ is larger when q is small. All this taken together, while increasing β and α both increase expected social welfare, increasing α has a greater impact on welfare than increasing β.

3 Extensions involving bias, precision, and information acquisition

In the introduction, I raised several issues on bias and precision, as well as how to endogenize q. The tractability of the quadratic utility function as well as the Gaussian noise terms make this game amenable to extensions that address these issues. In the earlier sections, I abstracted from the provider of public and private signals. In this section and the next, the provider of a particular signal becomes clear with context.

3.1 Endogenizing q through costly information acquisition with given precision

In Section 2, q was left as an exogenous parameter. However, it is possible to endogenize q in the model following the work of Arato et al. [2021]. To this end, consider a two-period game involving costly private information at a given precision level. As before, the agents only know that θ has an improper prior over $\mathbb{R}$, and there are two signals available: public and private. The public signal comes from the social planner, in the form of $y = \theta + \eta$ where $\eta \sim N\left(0, \frac{1}{\alpha}\right)$. On the other hand, the private signal comes from the media sector (separate from the social planner) in the form of $x_i = \theta + \varepsilon_i$ where $\varepsilon_i \sim N\left(0, \frac{1}{\beta}\right)$. The social planner provides public information indiscriminately, but the media sector only provides information to agents who pay some cost T.[9]

If agents decide to acquire private information, their expected utility is

$$\mathbb{E}_i^q\left[u_i\left(\boldsymbol{a}, \theta\right)\right] = -\mathbb{E}_i^q\left[\left(1-r\right)\left(a_{i,q} - \theta\right)^2 + r\left(L_i - \bar{L}\right)\right] - c_i\left(T\right), \tag{21}$$

while if agents decide to forego private information, their expected utility is

$$\mathbb{E}_i^{1-q}\left[u_i\left(\boldsymbol{a}, \theta\right)\right] = -\mathbb{E}_i^{1-q}\left[\left(1-r\right)\left(a_{i,1-q} - \theta\right)^2 + r\left(L_i - \bar{L}\right)\right]. \tag{22}$$

As before, $L_i = \int_0^1 \left(a_j - a_i\right)^2 dj$ and $\bar{L} = \int_0^1 L_j dj$. The cost of private information $c_i\left(T\right)$ is increasing in T. The index i indicates that the cost T may be perceived differently by each agent in the continuum.

At period $t = 0$, the social planner provides public information y and the media sector sets a price T. Agents decide whether or not to obtain the costly private information. At period $t = 1$, the information is provided by the media sector to those who have paid the amount T, and the agents choose their optimal actions $a_{i,q}$ or $a_{i,1-q}$ as in the earlier sections. Finally, utility and social welfare are realized. Unlike Arato et al. [2021], I do not have a subsidy term that returns the fees to the agents, as the fee is directly paid to the media sector. I do not model a game between the social planner and the media sector, as I focus on how

[9] One can think of the private signal as an aggregate of other normally distributed signals, and the cost T as an average cost to acquire the signal.

the measure q is obtained.[10]

The game is solved backwards. At $t = 1$, given the precisions of public and private information, the agents choose their optimal actions, and utility and social welfare are calculated as in Section 2. At $t = 0$, the agent must decide whether or not to obtain private information, given the precision of public information. The decision to incur the cost depends on the marginal benefit of having the information. In short, the agent compares the net expected utility from being informed compared to from not being informed. The agents solve the problem

$$\max_{I_i \in \{0,1\}} I_i \mathbb{E}_i^q \left[u_i \left(\boldsymbol{a}, \theta \right) \right] + \left[1 - I_i \right] \mathbb{E}_i^{1-q} \left[u_i \left(\boldsymbol{a}, \theta \right) \right] \tag{23}$$

where $I_i = 1$ when $\mathbb{E}_i^q \left[u_i \left(\boldsymbol{a}, \theta \right) \right] > \mathbb{E}_i^{1-q} \left[u_i \left(\boldsymbol{a}, \theta \right) \right]$ and 0 otherwise. In short, the agent will only pay to acquire private information that makes them better off in expectation.

Holding the action of agent i constant, the integral $\int_0^1 \left(a_j - a_i \right)^2 dj$ can be decomposed into the actions made by agents who decide to obtain private information $a_{j,q}$, and actions made by agents who decide to forego private information $a_{j,1-q}$, so that

$$L_i = \int_0^1 \left(a_j - a_i \right)^2 dj = \int_0^q \left(a_{j,q} - a_i \right)^2 dj + \int_q^1 \left(a_{j,1-q} - a_i \right)^2 dj \tag{24}$$

for some given q. Now consider an agent who decides to be informed. As before, denote their optimal action by $a_{i,q}$. Using (24), (21) can be written as

$$- \mathbb{E}_i^q \left[\left(1 - r \right) \left(a_{i,q} - \theta \right)^2 + r \left(\int_0^q \left(a_{j,q} - a_{i,q} \right)^2 dj + \int_q^1 \left(a_{j,1-q} - a_{i,q} \right)^2 dj - \bar{L} \right) - c_i \left(T \right) \right], \tag{25}$$

while (22) can be written as

$$- \mathbb{E}_i^{1-q} \left[\left(1 - r \right) \left(a_{i,1-q} - \theta \right)^2 + r \left(\int_0^q \left(a_{j,q} - a_{i,1-q} \right)^2 dj + \int_q^1 \left(a_{j,1-q} - a_{i,1-q} \right)^2 - \bar{L} \right) \right]. \tag{26}$$

The difference between these two expressions, which will be termed as the agent's value of private information, is

$$\mathbb{E}_i^q \left[u_{i,q} \left(\boldsymbol{a}, \theta \right) \right] - \mathbb{E}_i^{1-q} \left[u_{i,1-q} \left(\boldsymbol{a}, \theta \right) \right] \quad = \quad \frac{\beta \left(1 - r \right)^2 \left(\alpha + \beta \right)}{\alpha \left(\alpha + \beta \left(1 - rq \right) \right)^2} - c_i \left(T \right). \tag{27}$$

So there is a $c_i \left(T \right)$ for which agents would find it beneficial to acquire private information. Given that the parameters α, β, r and q are the same for all agents, a constant $c_i \left(T \right)$ for all $i \in [0, 1]$ would make it such

[10] I abstract from modelling the industrial organization of the media sector, and how prices are set, though a complete model would include the welfare of both the media entity and the continuum of agents, similar to Myatt and Wallace [2015], and would involve a game between the social planner and the media sector entities.

that either all agents would acquire information ($I_i = 1$) or no agent would acquire information ($I_i = 0$). Also, the agent's value of private information (27) is increasing in q, such that there is a q level for which the expression is exactly zero given some $c_i(T)$. Two points emerge from this. The first one is a multiple equilibria problem similar to the findings of Arato et al. [2021], but now it is in the acquisition of private information. Hellwig and Veldkamp [2009] state that multiplicity of equilibria will be absent if the information acquired was private, but that has proven to not be the case in this setting.

Second, the agent's value of private information is increasing in the measure of the agents that possess the information. This echoes the result in Proposition 1, and also the results of Hellwig and Veldkamp [2009] for actions with strategic complementarity - that agents would like to acquire information that others also have. The combination of strategic complementarity within the q-measure of agents, and the resulting common knowledge about the measure q of agents who would eventually be privately informed, results to an unstable equilibrium when (27) is equal to zero. The difference from Hellwig and Veldkamp [2009] may be due to the presence of private strategic complementarity for a subset of the agents in this model.

Arato et al. [2021] were able to eliminate the multiple equilibria problem by imposing an increasing pricing rule that depends on the measure of people who would like to obtain public information, which generates a strategic substitution effect. A simple way (in theory) to generate unique equilibria is to set $c_i(T)$ such that exactly a q- measure of agents value information to be not less than zero. In reality the effect of prices is relative to wealth - even the same price offered by the media sector would be perceived differently by agents owing to their wealth and income levels. As shown by Chiou and Tucker [2013], even though the information provided by a news outlet is the same for every individual that accesses the information, visits from individuals aged 18 to 24, and individuals who report an annual income below $30,000 per year dropped by 99 percent upon the installation of a paywall.

3.2 When it is costly to acquire more precise public and private information

Proposition 7 shows that the public information is more impactful than private information simply because of reach. In doing so I implicitly assumed that increasing α and β was costless. If information precision was costly, the policy implication would be to spend money on increasing the precision of public information, as it has the larger impact. This is shown formally here.

Let q be set exogenously. As before, let α and β represent precisions of public and private information, respectively. I abstract from the choice of α and β to focus on the social welfare effects of purchasing said precisions. Suppose the public and private precisions can be increased by paying $\tilde{c}$ and c per unit respectively. Thus, to obtain precision α costs $\tilde{c}\alpha$ while to obtain precision β costs $c\beta$.

The social planner decides on α, but spreads the costs such that each agent pays T_i, where $\int_0^1 T_i di = \tilde{c}\alpha$. On the other hand, suppose the precision β was jointly decided by the agents who were privately informed. The costs are spread to each privately-informed agent such that each of them pays $C_i(\beta) = \frac{c\beta}{q}$, and that $\int_0^q C_i(\beta)\, di = q\int_0^1 C_i(\beta)\, di = c\beta$. The agents who are not privately-informed have $C_i(\beta) = 0$.

With the preceding information, the agents' utility function is modified as follows:

$$\mathbb{E}_i\left[u_i(\boldsymbol{a}, \theta, \beta)\right] = -\mathbb{E}_i\left[(1-r)(a_i - \theta)^2 + r\left(L_i - \bar{L}\right) + (1-r)\left(C_i(\beta) + T_i\right)\right]. \tag{28}$$

Then, (normalized) expected social welfare is

$$\begin{aligned}\mathbb{E}(W|\theta) &= -\mathbb{E}\left(\int_0^1 \left((a_i-\theta)^2 + C_i(\beta) + T_i\right) di\right) \\ &= -q\left(\frac{\alpha\beta(1-r)^2 + (\alpha + \beta r(1-q))^2}{\alpha(\alpha + \beta(1-rq))^2} + \frac{1}{\alpha}\right) - \frac{1}{\alpha} - c\beta - \tilde{c}\alpha \end{aligned} \tag{29}$$

where $\int T_i di = \tilde{c}\alpha$ was used. As $q \to 0$, only the term $-\frac{1}{\alpha} - c\beta - \tilde{c}\alpha$ remains. This means that any effect of β will diminish along with q, yet the total cost of private precision stays at $c\beta$. On the other hand, an increase in α makes social welfare less negative. At $q = 0$, we have

$$\mathbb{E}(W|\theta, q=0) = -\frac{1}{\alpha} - c\beta - \tilde{c}\alpha \tag{30}$$

where social welfare actually diminishes with precision of private information β, because no one can use it. In contrast, whenever $\tilde{c} \leq \frac{1}{\alpha}$, expected social welfare is increasing in α. The results will not be different if I assume that $c = \tilde{c}$ as long as $c \leq \frac{1}{\alpha}$.

Colombo and Femminis [2008] use a similar setup to demonstrate that the acquisition of more precise public information crowds out that of private information if public information is cheaper than private information. While they show it in a multi-stage game where agents and the social planner decide on the precision of public and private information, I take the precision levels as given and consider what happens to social welfare for different cost specifications for public and private information precision, as $q \to 0$. Colombo and Femminis [2008] do this in a setting where all the agents have access to private and public information. It is probable that their results will be strengthened when only a subset of agents have access to private information, since public information becomes a stronger focal point in that case. In contrast, I use it not to obtain equilibrium levels of precision, but to inform the policy on which type of information to spend on when not all agents have access to public information.

3.3 What if information were biased rather than imprecise?

The baseline model was such that the signals obtained were all unbiased. However, in the age of widespread misinformation, it is possible that the public signal received by the agents is biased, or that agents believe that the public signals are biased despite not truly being so. Allcott and Gentzkow [2017] show that while only about 14 percent of Americans rely on social media as the primary source of news, a post-election survey that they conducted revealed that the average adult saw and remembered 1.14 fake stories during the campaign period for the 2016 elections. Cheli and Della Posta [2007] examine how bias and the perceptions about it affect the threshold level in a global game of currency speculation [Morris and Shin, 1998]. I use a similar approach to determine how bias affects social welfare.

There are three possible ways where the bias would appear. The first one is when there is bias in the public signal, but the agents are unaware that the bias exists. Next, there may be no bias in the public signal, but the agents perceive a bias. Finally, there is a bias in the public signal, which is correctly accounted for by the agents. In all cases, the bias is modelled as a constant $\gamma \neq 0$, which affects the relevant case.

3.3.1 Agents are unaware of a bias in the public signal

Let the model be as before, but let the public signal be of the form $y^b = \theta + \gamma + \eta$ where $\eta \sim N\left(0, \frac{1}{\alpha}\right)$ and $\gamma \neq 0$ is a constant. Suppose the agents are not aware that $\gamma \neq 0$ and so coordinate on the biased signal y^b. In other words, the agents will not correct for it. As such, the optimal action of an agent in the q-measure takes the form

$$a_{i,q}^b = (1-r)\,\mathbb{E}\left(\theta|x_i, y^b\right) + r\mathbb{E}\left(\bar{a}|x_i, y^b\right) = \frac{\beta(1-r)}{\alpha+\beta(1-rq)}x_i + \left(1 - \frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)(y+\gamma), \tag{31}$$

while the optimal action of some agent i in measure $1-q$ takes the form

$$a_{i,1-q}^b = (1-r)\,\mathbb{E}\left(\theta|y^b\right) + r\mathbb{E}\left(\bar{a}|y^b\right) = y^b = y+\gamma. \tag{32}$$

This means that the actions of the agents differ from the baseline case by a factor of γ. The action of the agents will pass through to expected social welfare. Let welfare under the use of biased public signal y^b be W^b, as follows:

$$\mathbb{E}\left(W^b|\theta\right) = -\mathbb{E}\left(\int_0^1 \left(a_i^b - \theta\right)^2 di|\theta\right) = \mathbb{E}\left(W|\theta\right) - \gamma^2\left[q\left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 + (1-q)\right] < \mathbb{E}\left(W|\theta\right) \tag{33}$$

As such, social welfare will be lower than in the baseline case.

3.3.2 Agents perceive a bias where none exists

Suppose the public signal was unbiased, i.e. the signal is of the form $y = \theta + \eta$ where $\eta \sim N\left(0, \frac{1}{\alpha}\right)$, but the agents believe that there is a bias term $\gamma \neq 0$. In that case, the agents will correct for the bias by subtracting it from y - i.e. they will use $y^a = y - \gamma$ in their decision-making. As such, the optimal action of an agent in the q-measure takes the form

$$a^a_{i,q} = (1-r)\,\mathbb{E}\left(\theta|x_i, y^a\right) + r\mathbb{E}\left(\bar{a}|x_i, y^a\right) = \frac{\beta\,(1-r)}{\alpha+\beta\,(1-rq)}x_i + \left(1 - \frac{\beta\,(1-r)}{\alpha+\beta\,(1-rq)}\right)(y-\gamma)\,, \tag{34}$$

while the optimal action of some agent i in measure $1-q$ takes the form

$$a^a_{i,1-q} = (1-r)\,\mathbb{E}\left(\theta|y^a\right) + r\mathbb{E}\left(\bar{a}|y^a\right) = y^a = y - \gamma. \tag{35}$$

This means that the agents' optimal actions "overcorrect" for something that isn't there, if they perceive a bias. This occurs when they do not trust the source of the public signal (the media, say). The overcorrection is carried through to expected social welfare. Let W^a indicate the welfare under the case where the agents overcorrect. Then, expected social welfare is

$$\mathbb{E}\left(W^a|\theta\right) = -\mathbb{E}\left(\int_0^1 \left(a^a_i - \theta\right)^2 di|\theta\right) = \mathbb{E}\left(W|\theta\right) - \gamma^2\left[q\left(\frac{\alpha+\beta r\,(1-q)}{\alpha+\beta\,(1-rq)}\right)^2 + (1-q)\right] < \mathbb{E}\left(W|\theta\right) \tag{36}$$

As in the case with agents who are unaware of the bias, social welfare is lower than that of the baseline case.

3.3.3 When agents are aware of an actual bias in the public signal

Let the public signal be of the form $y^b = \theta + \gamma + \eta$ where $\eta \sim N\left(0, \frac{1}{\alpha}\right)$ and $\gamma \neq 0$ is a constant. Suppose the agents are aware that $\gamma \neq 0$. Then, they will correct for the bias and adjust the biased signal y^b by γ - i.e. use $y^b - \gamma = y$. In that case, the actions will be the same as in the baseline case, and so will the expected social welfare.

Thus, when agents fail to correct a biased public signal, or when agents falsely correct an unbiased public signal, expected social welfare is lower than that of the baseline case. However, when agents correctly anticipate (and correct for) a biased public signal, expected social welfare is the same as in the baseline case. In short, when the agents' actions match the signals given out by the social planner, social welfare remains unchanged. However, when agents fail to correct for a bias in the signal given by the social planner, or when agents correct for a nonexistent bias, social welfare is reduced relative to the baseline case.

This has implications on how individuals' awareness of bias in news reporting affects their welfare. Newman et al. [2020, 2021] and Pew Research Center [2021b] report that while younger individuals mainly consume news through social media, they are aware of and are concerned about the misinformation that said news contains. This suggests that information is still desirable despite its biased nature, but agents must be aware of the bias for welfare to not be affected.[11]

4 Applications of the work

4.1 Shifting attitudes towards disease transmission and hygiene

Before the mid 19th century, disease transmission was not well-understood. The prevailing theory of transmission was miasma, or exposure to "bad air". As such, transmission through physical contact was not paid close attention to, and as a result, hygiene practices were not established in the medical profession. In Vienna, 1847, Ignaz Semmelweis found a link between mortality rates in maternity wards due to childbed fever and hygiene practiced by doctors. He instituted a strict regimen of handwashing with chlorinated lime within his own ward and subsequently drastically reduced mortality rates [Lane et al., 2010]. In London, 1854, John Snow was able to trace the cholera outbreak on Broad Street to a water pump that was contaminated by fecal material containing the cholera bacteria [Cameron and Jones, 1983]. This eventually led to the disabling of the aforementioned water pump and reform in water sanitation practices in the city. In Edinburgh, 1865, Joseph Lister used carbolic acid to create a barrier between the surgeon's hands and the patient's wounds, resulting to a drop in surgical mortality rates [Jessney, 2012]. Despite these successes, it took a long time for the medical profession to render the miasma theory obsolete, and also to institute handwashing protocols. It was not until the late 19th century that the miasma theory was debunked in favor of the germ theory, which is still used today [Halliday, 2001, Tesh, 1995].

In the aforementioned cases, the doctors had private knowledge which was very precise (high β), but found it difficult to spread their ideas to the general public (low q). While they were able to institute improvements to cases that were directly under their control, their work did not find immediate wide acceptance. That there was a prevailing theory meant that the general population would dismiss or ignore these cases. It may be the case that many more lives would have been saved had these measures been instituted sooner.

4.2 Reducing risky behaviors through television

It has been documented that multiple risk behaviors such as substance abuse and unprotected sexual activity cluster in adolescence [Kipping et al., 2012]. The social planner would prefer that agents know the

[11] This paper abstracts from the source of bias and why it is perpetuated. Gentzkow et al. [2015] model why biases occur in media, while Suen [2004] models why individuals who have biased views seek expert opinions from individuals who are similarly inclined.

consequences of the risk under consideration. Because of heterogeneity in educational styles and religious beliefs, some adolescents ($1 - q > 0$) are denied the knowledge that the social planner deems important. In the absence of public information, α is very low. As these behaviors have been associated with poor educational attainment and premature morbidity and mortality, they have been the target of various forms of interventions, such as reducing access, imposing punishments, and increasing information availability [Brooks et al., 2012, Hale and Viner, 2012, Jackson et al., 2012]. Public information campaigns belong to the latter. However, mere exposure to campaigns does not necessarily translate to desired changes in behavior. The National Youth Anti-Drug Media Campaign, a campaign that was mainly implemented through social marketing from 1998 to 2004, increased awareness but did not change behavior significantly [Hornik et al., 2008]. Similarly, the Safe Dates Project, which was a campaign to prevent dating violence which was mainly implemented through school and community channels, had mixed success - there was a change in both cognitive states and behavior one month after initial implementation. After a year, the cognitive risk factor effects were maintained, but the behavioral effects disappeared [Foshee et al., 1996, 1998, 2000]. As such, public information campaigns may not produce the desired change in behavior despite providing precise information.[12] On the other hand, engaging television shows may serve as effective public information campaigns, despite not intending to serve as such.[13] Kearney and Levine [2015] document a 4.3 percent reduction in teen pregnancy was found after the airing of "16 and Pregnant", a reality show by MTV. This echoes the findings of La Ferrara et al. [2012] that soap operas had a long run effect in the decline of the fertility rate in Brazil. More recently, Banerjee et al. [2019] document that male viewers of the MTV reality show "Shuga" were 21 percent less likely to justify violence than nonviewers in their sample. While DellaVigna and La Ferrara [2015] interpret the results of La Ferrara et al. [2012] and Kearney and Levine [2015] as the effects of imitation, their effective transmission of public signals on issues valued by the social planner cannot be overlooked.

4.3 The #MeToo movement

The MeToo movement started in 2006 when social activist Tarana Burke used the phrase "Me Too" on MySpace to empower through empathy women who have been subjected to sexual assault and harassment [Ohlheiser, 2017]. On October 16, 2017, amidst allegations against Hollywood producer Harvey Weinstein, actress Alyssa Milano tweeted, "If you've been sexually harassed or assaulted write 'me too' as a reply to this

[12] Weiss and Tschirhart [1994] outline what makes public information campaigns work, generally. See Wakefield et al. [2010] and Anker et al. [2016] for meta-analyses of studies that assess effectivity of mediated health campaigns.

[13] One possible reason for this is that when viewers are invested in television shows, they pay a lot of attention on it, thereby increasing the effective precision (in the sense of Myatt and Wallace [2012]) of any messages that may be transmitted through the show. This mechanism also increases the publicity factor of the message. In my model, this translates to a higher precision of public information α and degree of overall strategic complementarity r.

tweet." Within 24 hours of Milano's tweet, the hashtag had been tweeted nearly half a million times [Gilbert, 2017]. Between October 15, 2017 to September 30, 2018, the hashtag #MeToo was used about 19 million times [Anderson and Toor, 2018]. As in the image embedded within Milano's original tweet, the number of retweets and uses of the hashtag #MeToo gives an indicator of the magnitude of the problem.

In this example, prior to publicly using the hashtag, the individuals who had been victims of sexual assault and harassment are the ones who possess precise private information (high β) about being sexually assaulted and harassed. Other individuals would only be able to know about these issues when news outlets pick up cases, and news typically focuses on high profile cases. This selectivity could spread misconceptions about the severity of sexual assault [Evans, 2018] (low α, and potentially biased reporting). By tweeting #MeToo, agents obtain a sense of magnitude of how many other agents are privately informed, and can revise their actions accordingly. Social movements such as #MeToo not only inform agents about q, but the announcement itself also counts as a public signal. The use of hashtag activism itself has a crucial feature which is narrative agency - as these comments and retweets consist of numerous personal stories and appear in temporal order [Yang, 2016]. The narrative agency element in hashtag activism makes it different from news articles in that (multiple) tweets are more detailed, thus more informative (high α).

This has implications on the procedure for the selection of juries in cases involving sexual assault. While a person who possesses private information will act accordingly, a person who only possesses public information through the news may decide to act in a more lenient way. While a juror has to remain impartial, he must still be well-informed. If people keep their private information to themselves, and only few of them have this information, then it may be impossible to coordinate on punishing the accused.

4.4 Effect of media on consumer expectations

A number of macroeconomic models incorporate information rigidities among agents [Mankiw and Reis, 2002, Maćkowiak and Wiederholt, 2009, Sims, 2003, Woodford, 2003], assume that agents are ex ante homogeneous in their information structure or probability of obtaining new information. In practice, some agents, such as those employed by the central bank or large hedge funds are better than others in forecasting key macroeconomic variables. Recent academic work reveal that there seems to be little diffusion of specialized information to the rest of the population (only a measure $q < 1$ have private information x_i). Coibion and Gorodnichenko [2015] document that consumer expectations of inflation appear to fit the Phillips curve better than the those from professional forecasters and central bankers in the United States. More recently, Coibion et al. [2018] find that the forecasts of consumers are more aligned with the forecasts of firms compared to the forecasts of the professionals in New Zealand. Nonprofessional forecasters obtain their economic information from news reporting which is necessarily coarser than their source material. While news reports allow agents to coordinate their actions (high r), they also have a drop in precision (low α). In the context of my model, the professional forecasters could be interpreted as having both a private and a public signal, while nonprofessionals only have a public signal which is obtained from the media. The public signal received by the nonprofessionals translates into belief and behavioral changes, and may even lead to business cycles [Attanasio and Weber, 2010, Beaudry and Portier, 2014]. Eggers, Ellison, and Lee [2021] provide evidence that consumers are sensitive to recession announcements regardless of the severity of the recession, as the same information is available in official reports that are publicly available, and thus show that consumers do not use all available economic information.[14] Therefore the manner in which the media reports on economic figures matters for expectations.

4.5 Combating vaccine hesitancy

In 2019, the World Health Organization recognized vaccine hesitancy as a threat to global health.[15] During the 1980s, several countries that dropped their pertussis vaccinations following public protests regarding unfounded evidence against the diptheria-tetanus-pertussis (DTP) vaccine suffered 10 to 100 times higher incidence of pertussis following the vaccination [Gangarosa et al., 1998]. Previous incidences of vaccine hesitancy were traceable to vocal public figures covered by traditional forms of mass media. More recently, it has been found that vaccine hesitancy or outright refusal is more frequent in people who reported the internet and social media as their main source of information about vaccines [Dubé et al., 2021]. As social media is generally not monitored for veracity, it is a hotbed for clusters of individuals dubbed as "anti-vaxxers", who actively promote hoaxes and conspiracy theories against vaccines [Ahmed et al., 2020, Herrera-Peco et al., 2021b]. The presence of anti-vaxxers on a social network has been found to increase vaccine hesitancy (increase in q) [Wilson and Wiysonge, 2020].[16] The policy recommendation is the provision of correct and unified information (increase in α), but the manner of doing so requires fine-tuning as historical anti-vaccination movements have roots in the mistrust of the government [Hildreth and Alcendor, 2021, Poland and Jacobson, 2011, Wolfe and Sharp, 2002]. A more active role could be taken by health care professionals in propagating

[14] This may be due to the lack of expertise in the subject matter. Individuals such as central bankers and hedge fund managers typically have had several years of academic training and professional experience.

[15] The SAGE Working Group on Vaccine Hesitancy defines it as a delay in acceptance or refusal of vaccination despite availability of vaccination services. Individuals who are vaccine-hesistant may be reulctant but still accept some or all vaccines [MacDonald, 2015]. They are distinct from "anti-vaccine" individuals (anti-vaxxers) who actively oppose vaccination and may even spread misinformation regarding vaccines and their effectivity [Dubé et al., 2021]. Ther are also individuals who avoid vaccination for other reasons such as avoiding the revelation of their immigration status [Hildreth and Alcendor, 2021].

[16] Anti-vaxxers could be interpreted as receiving a biased private signal in addition to an unbiased public signal that they perceive as being biased. The agents who are influenced by the anti-vaxxers inherit the misconceptions possessed by the anti-vaxxers. Therefore, an increasing proportion of the population will act on misinformation, reducing social welfare. I do not model the propagation of q, but it is possible to motivate this within a network setting. For instance, Hsu et al. [2020] explore the decision of an agent to share their information across their network. The decision to share information is justified by having the agent's utility increase in as the beliefs of their followers become more aligned with theirs.

correct information on their personal social media [Herrera-Peco et al., 2021a]. Fact-checking, tweet-deletion and outright countering of false information by public health institutions may be effective mechanisms to counter conspiracy theories in social networks [Kauk et al., 2021, Steffens et al., 2019].

5 Conclusion

Individuals use information in proportion to their relative precision levels, regardless of whether information is public or private. When actions are strategic complements, public information becomes a coordination device, leading to the underuse of private information. Morris and Shin [2002] have shown that when all agents have both private and public information, excessive precision of public information is detrimental to social welfare. As such, research on the social value of public information has mainly focused on trying to mitigate the overuse of public information, or aligning the objectives of the social planner and the agents. This result no longer holds when access to private information is restricted to only a subset of agents, while maintaining uniform access to public information. Under this information structure, increasing the precision of both private and public information unconditionally increases social welfare.

The reason for this is strategic complementarity not only in actions but also in the use of information. In a beauty-contest setting, agents must calculate the average action chosen by everyone in the continuum. The knowledge that only a measure q of agents has access to private information has several effects: first, it reduces the proportion of agents who are able to use private information. At the same time, it creates a strategic complementarity within the subset of agents who have access to private information - they will only use private information to the extent that other people will do so as well. Yet they cannot ignore the overall strategic complementarity in actions. The interaction of q with the overall strategic complementarity measure r weakens the weight of private signals in individual actions for those who have them, leading to the strengthening of the public signal. Finally, this measure q acts as an aggregator of actions in the calculation of expected social welfare such that overall, private signals do not have a very strong impact.

Yet it is precisely the last point that makes increasing the precision of public information beneficial for social welfare. The reason is that while there is an inefficiency in that a q-measure of agents do not use their private information fully, the inefficiency is restricted to those agents. But all agents benefit from the increased focal power of public information, which increases in its precision. The loss of welfare from the underuse of private information is more than offset by the increase in welfare of the agents who do not have access to private information. This is in contrast to Morris and Shin [2002], who in assuming that all the agents have access to private information, also have a greater efficiency loss in that all the agents underuse their private information due to strategic complementarities. Thus, the result of Morris and Shin [2002] is a special case.

The results explain why public information campaigns remain a useful policy instrument despite research showing otherwise. We are at a time in history where simultaneously some individuals find it easier than ever to obtain information, while others have no means of doing so. For the ones who have access to private information, verifying the truth of the information remains a challenge. While it is possible to increase the precision of private information, doing so has limited positive impact on welfare simply because only a measure q of agents benefit from this. On the other hand, since in this setting, public signals have a stronger effect, increasing their precision goes a long way towards enhancing welfare.

6 Appendix

6.1 Calculations for Section 2

6.1.1 Obtaining $a_i = (1-r)\,\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{a})$

The expected utility of agent i is

$$\mathbb{E}_i\left[u_i(\boldsymbol{a},\theta)\right] = -\mathbb{E}_i\left[(1-r)(a_i-\theta)^2 + r\left(L_i - \bar{L}\right)\right]. \tag{37}$$

Take the partial derivative of $\mathbb{E}_i\left[u_i(\boldsymbol{a},\theta)\right]$ with respect to a_i and equate to 0 to obtain first-order condition

$$\frac{\partial \mathbb{E}_i\left[u_i(\boldsymbol{a},\theta)\right]}{\partial a_i} = -\mathbb{E}_i\left[(1-r)\,2(a_i-\theta) - 2r(\bar{a}-a_i)\right] = 0 \tag{38}$$

where note that $\frac{\partial L_i}{\partial a_i} = -2(\bar{a}-a_i)$ from writing

$$L_i = \int_0^1 (a_j - a_i)^2\, dj = \int_0^1 a_j^2 dj - 2a_i \int_0^1 a_j dj + \int_0^1 a_i^2 dj = \int_0^1 a_j^2 dj - 2a_i\bar{a} + a_i^2. \tag{39}$$

Solving for a_i in (38) yields $a_i = (1-r)\,\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{a})$.

6.1.2 Average action $\bar{a}$

Since all agents know the information structure across all agents, they all calculate the same average action $\bar{a}$ taking the measures q and $1-q$ of agents:

$$
\begin{aligned}
\bar{a} &= q\int_0^1 a_{j,q}dj + (1-q)\int_0^1 a_{j,1-q}dj \\
&= q\left[\int_0^1 \left(\kappa x_j + (1-\kappa)\,y\right)\right] + (1-q)\,y \qquad (40)\\
&= q\left[\kappa\int_0^1 (\theta+\varepsilon_j) + (1-\kappa)\,y\right] + (1-q)\,y \\
&= q\kappa\theta + (1-q\kappa)\,y.
\end{aligned}
$$

6.1.3 Proof that $a_{i,1-q} = y$

Some agent i who only obtains signal y will have optimal action $a_{i,1-q}$ of the form

$$
a_{i,1-q} = (1-r)\,\mathbb{E}_{i,1-q}(\theta) + r\mathbb{E}_{i,1-q}(\bar{a}). \qquad (41)
$$

Plugging $\bar{a}$ into into the optimal action $a_{i,1-q}$ yields

$$
\begin{aligned}
a_{i,1-q} &= (1-r)\,\mathbb{E}_{i,1-q}(\theta) + r\mathbb{E}_{i,1-q}(\bar{a}) \\
&= (1-r)\,\mathbb{E}_{i,1-q}(\theta) + r\mathbb{E}_{i,1-q}\left(q\kappa\theta + (1-q\kappa)\,y\right) \qquad (42)\\
&= (1-r+rq\kappa)\,y + r\,(1-q\kappa)\,y \\
&= y,
\end{aligned}
$$

which is what I set out to prove. The penultimate line stems from the fact that since indeed the only signal that the $1-q$ subset of agents obtain is y, $\mathbb{E}_i^{1-q}(\theta) = \mathbb{E}(\theta|y) = y$.

6.1.4 Proof that $\kappa = \frac{\beta(1-r)}{\alpha+\beta(1-rq)}$ for the agents in group q

For agents who have both private and public signals, the first-order condition is $a_{i,q} = (1-r)\,\mathbb{E}_{i,q}(\theta) + r\mathbb{E}_{i,q}(\bar{a})$. Using $\bar{a}$ yields

$$
\begin{aligned}
a_{i,q} &= (1-r)\,\mathbb{E}_{i,q}(\theta) + r\mathbb{E}_{i,q}(\bar{a}) \\
&= (1-r)\,\mathbb{E}_{i,q}(\theta) + r\mathbb{E}_{i,q}\left(q\kappa\theta + (1-q\kappa)\,y\right) \qquad (43)\\
&= (1-r\,(1-q\kappa))\,\mathbb{E}_{i,q}(\theta) + r\,(1-q\kappa)\,y \\
&= \frac{\beta\,(1-r+rq\kappa)}{\alpha+\beta}x_i + \left[1-\frac{\beta\,(1-r+rq\kappa)}{\alpha+\beta}\right]y
\end{aligned}
$$

using the fact that $\mathbb{E}_{i,q}(\theta) = \mathbb{E}(\theta|x_i, y) = \frac{\beta x_i + \alpha y}{\alpha+\beta}$. Recall also that for these agents, the optimal action is of the form $a_{i,q} = \kappa x_i + (1-\kappa) y$. Matching coefficients to $a_{i,q} = \kappa x_i + (1-\kappa) y$ and solving for κ, yields $\kappa = \frac{\beta(1-r)}{\alpha+\beta(1-rq)}$.

6.1.5 Expected social welfare derivation

Following Cornand and Heinemann [2008], expected social welfare can be calculated by splitting the continuum of agents into the ones who are privately-informed and the ones who are not:

$$\begin{aligned}
\mathbb{E}(W|\theta) &= -\mathbb{E}\left(\int_0^1 (a_i - \theta)^2 \, di|\theta\right) \\
&= -\mathbb{E}\left(q\int_0^1 (a_{i,q} - \theta)^2 \, di + (1-q)\int_0^1 (a_{i,1-q} - \theta)^2 \, di|\theta\right) \\
&= -\mathbb{E}\left(q\int_0^1 \left(\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\varepsilon_i + \left(1 - \frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)\eta\right)^2 di + (1-q)\int_0^1 (\eta)^2 \, di|\theta\right) \quad (44) \\
&= -\left[q\int_0^1 \left(\left(\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)^2 \frac{1}{\beta} + \left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 \frac{1}{\alpha}\right) di + \frac{1-q}{\alpha}\right] \\
&= -q\left(\frac{\alpha\beta(1-r)^2 + (\alpha+\beta r(1-q))^2}{\alpha(\alpha+\beta(1-rq))^2}\right) - \left(\frac{1-q}{\alpha}\right).
\end{aligned}$$

6.2 Details on endogenizing q

Arato et al. [2021] endogenized the fraction of users who should have access to public information in order to optimize expected social welfare. I apply the same procedure to find the fraction of users who should have access to private information.

At $t = 0$, agents solve the problem

$$\begin{aligned}
&\max_{I_i \in \{0,1\}} I_i \mathbb{E}_i^q [u_i(\boldsymbol{a}, \theta)] + [1 - I_i] \mathbb{E}_i^{1-q} [u_i(\boldsymbol{a}, \theta)] \quad (45) \\
&= \max_{I_i \in \{0,1\}} \mathbb{E}_i^{1-q} [u_i(\boldsymbol{a}, \theta)] + I_i \left\{\mathbb{E}_i^q [u_i(\boldsymbol{a}, \theta)] - \mathbb{E}_i^{1-q} [u_i(\boldsymbol{a}, \theta)]\right\}
\end{aligned}$$

where $I_i = 1$ when $\mathbb{E}_i^q [u_i(\boldsymbol{a}, \theta)] > \mathbb{E}_i^{1-q} [u_i(\boldsymbol{a}, \theta)]$ and 0 otherwise. In short, the agent will only pay to acquire private information that makes them better off in expectation.

If agents decide to acquire private information, their expected utility is

$$\mathbb{E}_i^q [u_i(\boldsymbol{a}, \theta)] = -\mathbb{E}_i^q \left[(1-r)(a_{i,q} - \theta)^2 + r\left(L_i - \bar{L}\right)\right] - c_i(T), \quad (46)$$

while if agents decide to forego private information, their expected utility is

$$\mathbb{E}_i^{1-q}\left[u_i\left(\boldsymbol{a},\theta\right)\right] \quad = \quad -\mathbb{E}_i^{1-q}\left[\left(1-r\right)\left(a_{i,1-q}-\theta\right)^2+r\left(L_i-\bar{L}\right)\right]. \tag{47}$$

As before, $L_i = \int_0^1 \left(a_j - a_i\right)^2 dj$ and $\bar{L} = \int_0^1 L_j dj$. Holding the action of agent i constant, the integral $\int_0^1 \left(a_j - a_i\right)^2 dj$ can be decomposed into the actions made by agents who decide to obtain private information $a_{j,q}$, and actions made by agents who decide to forego private information $a_{j,1-q}$, so that

$$L_i = \int_0^1 \left(a_j - a_i\right)^2 dj = \int_0^q \left(a_{j,q} - a_i\right)^2 dj + \int_q^1 \left(a_{j,1-q} - a_i\right)^2 dj \tag{48}$$

for some given q.

Recall that in equilibrium, given q, any informed agent i will have an optimal action of the form $a_{i,q} = \kappa x_i + \left(1-\kappa\right)y$, where $\kappa = \frac{\beta(1-r)}{\alpha+\beta(1-rq)}$, and any uninformed agent i will have an optimal action of the form $a_{j,1-q} = y$. The following forms follow:

$$\begin{aligned}
\left(a_{i,q}-\theta\right)^2 = \left(\kappa\varepsilon_i + \left(1-\kappa\right)\eta\right)^2 \quad &\Rightarrow \quad \mathbb{E}_i^q\left(a_{i,q}-\theta\right)^2 = \frac{\kappa^2}{\beta} + \frac{\left(1-\kappa\right)^2}{\alpha} \\
\left(a_{j,q}-a_{i,q}\right)^2 = \kappa^2\left(\varepsilon_j-\varepsilon_i\right)^2 \quad &\Rightarrow \quad \mathbb{E}_i^q\left(a_{j,q}-a_{i,q}\right)^2 = \frac{2\kappa^2}{\beta} \\
\left(a_{j,1-q}-a_{i,q}\right)^2 = \kappa^2\left(\eta-\varepsilon_i\right)^2 \quad &\Rightarrow \quad \mathbb{E}_i^q\left(a_{j,1-q}-a_{i,q}\right)^2 = \frac{\kappa^2}{\alpha+\beta} \\
\left(a_{i,1-q}-\theta\right)^2 = \eta^2 \quad &\Rightarrow \quad \mathbb{E}_i^{1-q}\left(a_{i,1-q}-\theta\right)^2 = \frac{1}{\alpha} \\
\left(a_{j,q}-a_{i,1-q}\right)^2 = \kappa^2\left(\varepsilon_j-\eta\right)^2 \quad &\Rightarrow \quad \mathbb{E}_i^{1-q}\left(a_{j,q}-a_{i,1-q}\right)^2 = \frac{\kappa^2}{\alpha+\beta} \\
\left(a_{j,1-q}-a_{i,1-q}\right)^2 = 0 \quad &\Rightarrow \quad \mathbb{E}_i^{1-q}\left(a_{j,1-q}-a_{i,1-q}\right)^2 = 0
\end{aligned} \tag{49}$$

Substituting (49) into (46) and (47), we have

$$\begin{aligned}
&\mathbb{E}_i^q\left[u_{i,q}\left(\boldsymbol{a},\theta\right)\right] \\
&= -\,\mathbb{E}_i^q\left[\left(1-r\right)\left(a_{i,q}-\theta\right)^2 + r\left(\int_0^q \left(a_{j,q}-a_{i,q}\right)^2 dj + \int_q^1 \left(a_{j,1-q}-a_{i,q}\right)^2 dj - \bar{L}\right) - c_i\left(T\right)\right] \\
&= -\left(1-r\right)\left[\frac{\kappa^2}{\beta} + \frac{\left(1-\kappa\right)^2}{\alpha}\right] - r\left[\frac{2q\kappa^2}{\beta} + \frac{\left(1-q\right)\kappa^2}{\alpha+\beta}\right] - r\bar{L} - c_i\left(T\right)
\end{aligned} \tag{50}$$

and

$$\begin{aligned}
&\mathbb{E}_i^{1-q}\left[u_{i,1-q}\left(\boldsymbol{a},\theta\right)\right] \\
=&-\mathbb{E}_i^{1-q}\left[\left(1-r\right)\left(a_{i,1-q}-\theta\right)^2+r\left(\int_0^q\left(a_{j,q}-a_{i,1-q}\right)^2dj+\int_q^1\left(a_{j,1-q}-a_{i,1-q}\right)^2dj-\bar{L}\right)-c_i\left(T\right)\right] \quad (51)\\
=&-\frac{\left(1-r\right)}{\alpha}-\frac{rq\kappa^2}{\alpha+\beta}-r\bar{L}.
\end{aligned}$$

Thus,

$$\begin{aligned}
&\mathbb{E}_i^{q}\left[u_{i,q}\left(\boldsymbol{a},\theta\right)|\theta\right]-\mathbb{E}_i^{1-q}\left[u_{i,1-q}\left(\boldsymbol{a},\theta\right)\right] \\
=\;&-\left(1-r\right)\left[\frac{\kappa^2}{\beta}+\frac{\left(1-\kappa\right)^2}{\alpha}\right]-r\left[\frac{2q\kappa^2}{\beta}+\frac{\left(1-q\right)\kappa^2}{\alpha+\beta}\right]-c_i\left(T\right)-\left[-\frac{\left(1-r\right)}{\alpha}-\frac{rq\kappa^2}{\alpha+\beta}\right] \\
=\;&\frac{1}{\beta}\left[-\left(1-r\right)\kappa^2-2rq\kappa^2-r\left(1-q\right)\kappa^2+rq\kappa^2\right]+\frac{1}{\alpha}\left[-\left(1-r\right)\left(1-\kappa\right)^2-r\left(1-q\right)\kappa^2+\left(1-r\right)+rq\kappa^2\right]-c_i\left(T\right) \\
=\;&\frac{-\kappa^2}{\beta}+\frac{1}{\alpha}\left[\left(1-r\right)\left(1-\left(1-\kappa\right)^2\right)-r\kappa^2\left(1-2q\right)\right]-c_i\left(T\right) \\
=\;&\frac{\beta\left(1-r\right)^2\left(\alpha+\beta\right)}{\alpha\left(\alpha+\beta\left(1-rq\right)\right)^2}-c_i\left(T\right)
\end{aligned}$$

where the final line comes from using $\kappa=\frac{\beta(1-r)}{\alpha+\beta(1-rq)}$.

6.3 Details on calculating expected social welfare with costly acquisition of precise information

$$\begin{aligned}
\mathbb{E}\left(W|\theta\right) &= -\mathbb{E}\left(\int_0^1\left(\left(a_i-\theta\right)^2+C_i\left(\beta\right)+T_i\right)di\right) \\
&= -\mathbb{E}\left(\int_0^1\left(\left(a_i-\theta\right)^2+C_i\left(\beta\right)+T_i\right)di\right) \\
&= -\mathbb{E}\left(\int_0^q\left(\left(a_{i,q}-\theta\right)^2+\frac{c\beta}{q}+T_i\right)di+\int_q^1\left(\left(a_{i,1-q}-\theta\right)^2+T_i\right)di\right) \\
&= -\left[q\left(\left(\frac{\beta\left(1-r\right)}{\alpha+\beta\left(1-rq\right)}\right)^2\frac{1}{\beta}+\left(\frac{\alpha+\beta r\left(1-q\right)}{\alpha+\beta\left(1-rq\right)}\right)^2\frac{1}{\alpha}+\frac{c\beta}{q}+\tilde{c}\alpha\right)+\left(1-q\right)\left(\frac{1}{\alpha}+\tilde{c}\alpha\right)\right] \\
&= -q\left(\frac{\alpha\beta\left(1-r\right)^2+\left(\alpha+\beta r\left(1-q\right)\right)^2}{\alpha\left(\alpha+\beta\left(1-rq\right)\right)^2}\right)-\left(\frac{1-q}{\alpha}\right)-c\beta-\tilde{c}\alpha \\
&= -q\left(\frac{\alpha\beta\left(1-r\right)^2+\left(\alpha+\beta r\left(1-q\right)\right)^2}{\alpha\left(\alpha+\beta\left(1-rq\right)\right)^2}+\frac{1}{\alpha}\right)-\frac{1}{\alpha}-c\beta-\tilde{c}\alpha
\end{aligned}$$

6.4 Details on the calculation of social welfare under different assumptions involving bias

6.4.1 Expected welfare calculation when agents are unaware of a bias in the public signal

$$
\begin{aligned}
\mathbb{E}\left(W^b|\theta\right) &= -\mathbb{E}\left(\int_0^1 \left(a_i^b-\theta\right)^2 di|\theta\right) \\
&= -\mathbb{E}\left(q\int_0^1 \left(a_{i,q}^b-\theta\right)^2 di + (1-q)\int_0^1 \left(a_{i,1-q}^b-\theta\right)^2 di|\theta\right) \\
&= -\mathbb{E}\left(q\int_0^1 \left(\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\varepsilon_i + \left(1-\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)(\eta+\gamma)\right)^2 di + (1-q)\int_0^1 (\eta+\gamma)^2 di\right) \\
&= -\left[q\int_0^1 \left(\left(\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)^2 \frac{1}{\beta} + \left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 \frac{1}{\alpha} + \left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 \gamma^2\right) di\right] \\
&\quad -\frac{1-q}{\alpha} - (1-q)\gamma^2 \\
&= -q\left(\frac{\alpha\beta(1-r)^2 + (\alpha+\beta r(1-q))^2}{\alpha(\alpha+\beta(1-rq))^2}\right) - \frac{1-q}{\alpha} - \gamma^2\left[q\left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 + (1-q)\right] \\
&< \mathbb{E}(W|\theta)
\end{aligned}
$$

6.4.2 Expected welfare calculation when agents perceive a bias in the public signal where none exists

$$
\begin{aligned}
\mathbb{E}\left(W^a|\theta\right) &= -\mathbb{E}\left(\int_0^1 \left(a_i^a-\theta\right)^2 di|\theta\right) \\
&= -\mathbb{E}\left(q\int_0^1 \left(a_{i,q}^a-\theta\right)^2 di + (1-q)\int_0^1 \left(a_{i,1-q}^a-\theta\right)^2 di|\theta\right) \\
&= -\mathbb{E}\left(q\int_0^1 \left(\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\varepsilon_i + \left(1-\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)(\eta-\gamma)\right)^2 di + (1-q)\int_0^1 (\eta-\gamma)^2 di\right) \\
&= -\left[q\int_0^1 \left(\left(\frac{\beta(1-r)}{\alpha+\beta(1-rq)}\right)^2 \frac{1}{\beta} + \left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 \frac{1}{\alpha} + \left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 \gamma^2\right) di\right] \\
&\quad -\frac{1-q}{\alpha} - (1-q)\gamma^2 \\
&= -q\left(\frac{\alpha\beta(1-r)^2 + (\alpha+\beta r(1-q))^2}{\alpha(\alpha+\beta(1-rq))^2}\right) - \frac{1-q}{\alpha} - \gamma^2\left[q\left(\frac{\alpha+\beta r(1-q)}{\alpha+\beta(1-rq)}\right)^2 + (1-q)\right] \\
&< \mathbb{E}(W|\theta)
\end{aligned}
$$

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Figure 1: $\mathbb{E}\left(W|\theta\right)$ as a function of q for various values of α,β, and r

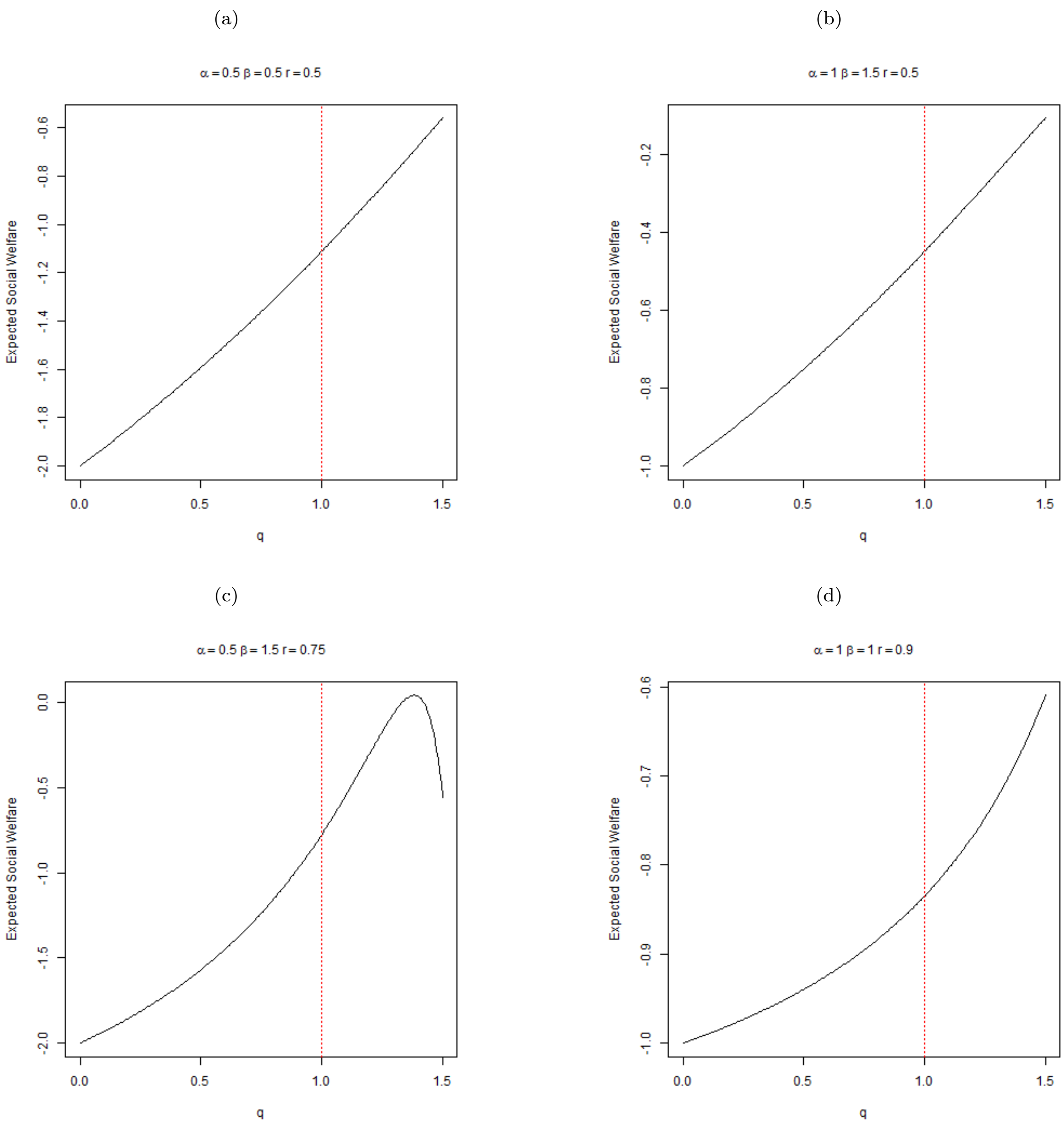

Figure 2: $\mathbb{E}\left(W|\theta\right)$ in (α, β) space for various values of r and q