Belief and Higher-Order Belief in the Centipede Games: Theory and Experiment*

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Abstract

This paper experimentally explores people's beliefs behind the failure of backward induction in the centipede games. I elicit players' beliefs about opponents' strategies and 1st-order beliefs. I find that subjects maximize their monetary payoffs according to their stated beliefs less frequently in the Baseline Centipede treatment where an efficient non-equilibrium outcome exists; they do so more frequently in the Constant Sum treatment where the efficiency property is removed. Moreover, subjects believe their opponents' maximizing behavior and expect their opponents to hold the same belief less frequently in the Baseline Centipede treatment and more frequently in the Constant Sum treatment.

Keywords: Centipede Game; Rationality; Belief and Higher Order Belief; Laboratory Experiments; Learning

JEL Classification: C72; C92; D83

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1 Introduction

The "backward induction paradox" has long been a challenge to the solution concept in perfect-information extensive-form games. The centipede game, introduced by Rosenthal (1981), is a classical example of the paradox. One may simply attribute people's failure of playing a subgame perfect equilibrium to an inability to think rationally and to calculate payoffs correctly; nevertheless, it is possible that people indeed maximize expected payoffs given certain beliefs about their opponent's strategy and level of rationality. In other words, people are not making mistakes as often as they are thought to be; the gap between experimental observations and the theoretical prediction stems from people's inconsistent beliefs and higher-order beliefs.

This paper studies rationality, belief of rationality, and higher-order belief of rationality in the centipede game experiment. Actual play in centipede experiments seldom ends as backward induction predicts. The existing literature attributes the departure from backward induction (BI thereafter) prediction either to players' lack of rationality, or to players' inconsistent beliefs and higher-order beliefs of others' rationality. In this paper, we evaluate these arguments in a more direct fashion. We elicit the first mover's belief about the second mover's strategy as well as the second mover's initial and conditional beliefs about the first mover's strategy and belief. The measured beliefs help us to infer the conditional probability systems (CPS thereafter) of both players. The inferred CPS's and players' actual strategy choices identify *why* they fail to reach the BI outcomes.

The first strand of the existing experimental literature focuses on players' lack of rationality. It presumes the presence of behavioral types who fail to or do not maximize monetary payoffs[1]. For example, McKelvey and Palfrey (1992) assume that ex-ante a player chooses to not play along the BI path with probability p. But assuming irrationality before a game starts is restrictive; people could be right but think others are wrong. In this paper, the inferred CPS and players' strategies allow us to directly examine players' rationality. We define rationality as a player's strategy best responding to his or her measured belief. We find, in both experimental treatments, that the frequency of either player being rational is less than 100 percent but not significantly below 80 percent. Moreover, in the Constant-Sum treatment, which excludes players' efficiency preferences as a confounding factor, the frequency of both players being rational is significantly higher than that in the classical centipede game treatments.

[1]See McKelvey and Palfrey (1992), Fey et.al.(1996), Zauner (1999), Kawagoe and Takizawa (2012).

Another strand of literature attributes the experimental anomalies to a lack of common knowledge of rationality. Two field centipede experiments (Palacios-Huerta and Volij (2009) and Levitt et.al (2009)) are in this fashion. Both use professional chess players as experimental subjects; the authors assume there is always rationality and common knowledge of rationality among chess players. The authors' approach is based on Aumann's (1995) claim[2] "if common knowledge of rationality holds then the backward induction outcome results." Nevertheless, the notion of "common knowledge" is not empirically verifiable; one can never ensure the existence of "common knowledge" among chess-players or the non-existence of it among ordinary laboratory subjects. Instead, it is the players' beliefs that matter in the actual play. Arguments that based on players' "knowledge" may have limited explanatory power for the anomalies in the centipede experiments. Thus in this paper we follow an alternative approach, the belief-based epistemic game theory[3] to address the notion of common *belief* of rationality. The measured beliefs, high-order beliefs, and players' actual strategy choices help us to identify whether *rationality and common initial belief of rationality* and/or *rationality and common strong belief of rationality* hold.

We find, in fact, that even common *initial* belief of rationality does not exist in the laboratory. In both experimental treatments, the frequency of the players believing in their opponents' rationality is less than 100 percent. Nevertheless, in the Constant-Sum treatment this frequency is significantly higher than that in the Baseline Centipede treatments, a treatment that bears an efficiency property for the sum of both players' payoffs. Moreover, in both treatments the average frequency of the second mover's initial belief of the first mover's rationality and 2nd-order rationality is less than 100 percent. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatments. Also it gradually increases toward 100 percent as subjects gain experience in later rounds of the experiment. By contrast, in the Baseline centipede treatment there is no such increasing pattern as more rounds are played.

Furthermore, we find that common *strong* belief of rationality is seldom observed in the laboratory, especially for the second-mover. In both treatments, the average frequency of the second mover's strongly believing in the first-mover's rationality and 2nd-order rationality is significantly less than 100 percent. And this frequency in the Constant-Sum treatment does not significantly differ from those in the other two treatments. Notice that the second-movers are informed that the first-mover has chosen a non-BI strategy for the

[2]For more knowledge-based theoretical discussion on the "backward induction paradox," see Bicchieri (1988, 1989), Pettit and Sugden (1989), Reny (1988, 1992), Bonanno (1991), Aumann (1995, 1996, 1998), Binmore (1996, 1997).

[3]See Aumann and Brandenburger (1995), Battigalli (1997), Battigalli and Siniscalchi (1999, 2002), Ben-Porath (1997), Brandenburger (2007).

first stage before being asked to state their conditional beliefs. Thus our result indicates that once the second-movers observe the first-movers' deviating from the BI path, the former can hardly believe in the latter's rationality AND higher-order belief in rationality.

We consider a model with uncertainty in players' payoff types to account for the treatment effects on players' beliefs and higher-order beliefs. In this model, not only rationality and higher-order rationality, but also players' payoff types are not common-knowledge. The model allows for incomplete information regarding players' payoffs: a portion of the players are efficiency-oriented and derive utility from the larger sum of his/her and opponent's payoffs. In this model the notion of *rationality and common initial/strong belief of rationality* imposes less restrictive conditions on both players' beliefs. The epistemic states that satisfy certain restrictions on players' beliefs involves a richer set of conditional probability systems for both players. The efficiency property of the classical centipede game diffuses players' beliefs and higher order beliefs, which in turn contribute to players' non-equilibrium strategy choices observed in most previous centipede experiments.

The remainder of the paper is organized as follows. Section 2 formally defines players' beliefs, rationality, and beliefs of rationality in the centipede game. Section 3 presents the experimental design in detail, with Section 3.1 introducing experimental treatments and testing hypothesis and Section 3.2 introducing the procedure and belief elicitation method in the laboratory. Section 4 presents the experimental findings on players' strategies, players' beliefs about opponents' strategies, rationality, and higher-order beliefs of rationality. Section 5 presents a model with efficiency-oriented players to account for the observed treatment effect. Section 6 reviews related theoretical literature on backward induction and epistemic game theory and previous experimental studies on the centipede games. Section 7 concludes.

2 Defining Belief, Rationality, and Belief of Rationality

To capture the infinite hierarchy of players' beliefs, the epistemic game theory enrich the classical formulation of a game by adding set of belief *types* for the players (Harsanyi 1967). We follow Brandenburger's [15] notation of players' belief types and epistemic states throughout this section. Denote the two-player (Ann and Bob) finite centipede game $\langle S^a, S^b, \Pi^a, \Pi^b \rangle$ where S^i and Π^i represent player i's set of pure strategies and set of payoffs, respectively.

Definition 1. *We call the structure* $\langle S^a, S^b; T^a, T^b; \lambda^a(\cdot), \lambda^b(\cdot)\rangle$ *a type structure for the players of a two-person finite game where* T^a *and* T^b *are compact metrizable space, and each* $\lambda^i : T^i \to \Delta(S^{-i} \times T^{-i}), i = a, b$ *is continuous. An element* $t^i \in T^i$ *is called a* ***type*** *for player* $i, (i = a, b)$. *An elements* $(s^a, s^b, t^a, t^b) \in S \times T$ *(where* $S = S^a \times S^b$ *and* $T = T^a \times T^b$*) is called a* ***state****.*

We first define *rationality* using the type-state language:

Definition 2. *A strategy-type pair of player* $i, (i = a, b)$, (s^i, t^i) *is* ***rational*** *if* s^i *maximizes player i's expected payoff under the measure* $\lambda^i(t^i)$*'s marginal on* S^{-i}*.*

Next, we define a player's believing an event as:

Definition 3. *Player i's type* t^i *believes an event* $E \subseteq S^{-i} \times T^{-i}$ *if* $\lambda^i(t^i)(E) = 1, i = a, b$. *Denote*

$$B^i(E) = \{t^i \in T^i : t^i \text{ believes } E\}, i = a, b$$

the set of player i's types that believe the event E*.*

For each player i, denote R_1^i the set of all rational strategy-type pairs (s^i, t^i). Thus R_1^{-i} stands for the set of all rational strategy-type pairs of opponent $-i$, i.e.

$$R_1^{-i} = \{(s^{-i}, t^{-i}) \in S^{-i} \times T^{-i} : (s^{-i}, t^{-i}) \text{ is rational.}\}, i = a, b$$

Now we can define a player's believing in his or her opponent's rationality as player i believes the event $E = R_1^{-i}$:

Definition 4. *Player i's type* t^i ***believes his or her opponent's rationality*** $R_1^{-i} \subseteq S^{-i} \times T^{-i}$ *if* $\lambda^i(t^i)(R_1^{-i}) = 1$. *Denote*

$$B^i(R_1^{-i}) = \{t^i \in T^i : t^i \text{ believes } R_1^{-i}\}, i = a, b$$

the set of player i's types that believe opponent $-i$*'s rationality.*

Then for all $m \in \mathbf{N}$ and $m > 1$, we can define R_m^i inductively by

$$R_m^i = R_{m-1}^i \cap (S^i \times B^i(R_{m-1}^{-i})), i = a, b$$

And write $R_m = R_m^a \times R_m^b$. Then players' higher order beliefs of rationality are defined in the following way:

Definition 5. *If a state* $(s^a, s^b, t^a, t^b) \in R_{m+1}$*, we say that there is* ***rationality and*** m***th-order belief of rationality*** *(RmBR) at this state.*
If a state $(s^a, s^b, t^a, t^b) \in \cap_{m=1}^{\infty} R_m$*, we say that there is* ***rationality and common belief of rationality*** *(RCBR) at this state.*

For a perfect-information sequential move game such as the centipede game, in case the game situation involves the players not playing the backward-induction path (BI path thereafter), we also need to describe each player's belief at a probability-0 event. We use the tool of *conditional probability systems* (CPS thereafter) introduced by Renyi [32]. It consists of a family of conditional events and one probability measure for each of these events. For the centipede game under analysis, we say that player i ***initially believes event E*** if i's CPS assigns probability 1 to event E at the *root* of the perfect-information game tree. We denote the set of player i's types that initially believe event E as $\mathrm{IB}^i(E), i = a, b$. We also say that player i ***strongly believes event E*** if for any information set H that is reached, i.e. $E \cap (H \times T^{-i}) \neq \emptyset$, i's CPS assigns probability 1 to event E. We denote the set of a player's types who strongly believe event E as $\mathrm{SB}^i(E), i = a, b$.

3 Experimental Design

We experimentally investigate players' rationality, beliefs and higher-order beliefs about opponents' rationality. Section 3.1 describes the treatments and hypotheses. Section 3.2 details the laboratory environments, belief elicitation, and other experimental procedures.

3.1 Treatments and Hypotheses

Our experiment consists of two treatments, each of which is a three-legged centipede game. The first treatment, "Baseline Centipede Game" is shown in Figure 1. Both the Nash equilibrium outcome and Subgame Perfect equilibrium involve player A choosing OUT at the first stage and the two players ending up with a $20 - 10$ split of payoffs.

Here we emphasize the *efficiency property* of this baseline game: first, in accordance with the "backward induction paradox" discussed in the theoretical literature, the sum of

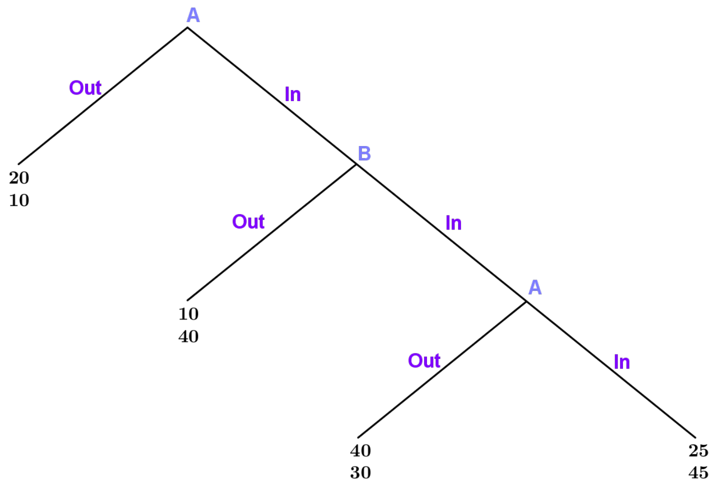


Figure 1: Baseline Centipede Game

the players' payoffs grows at each stage. Had the players not played the BI strategy and reached the last stage, the outcome of the game would yield both players higher payoffs than the Nash equilibrium outcome. So the non-BI outcome is *efficient*.

Suppose for the moment that players only maximize their own monetary payoffs. In the baseline game it is easy to calculate the players' cutoff beliefs in support of their pure strategies. If player A expects player B to play IN with a probability greater than $\frac{1}{3}$, then A's best response is to play IN for the first stage and OUT for the third stage. As for player B, if he/she expects player A to play IN at the third stage with a probability greater than $\frac{2}{3}$, then B's best response is to play IN for the second stage.

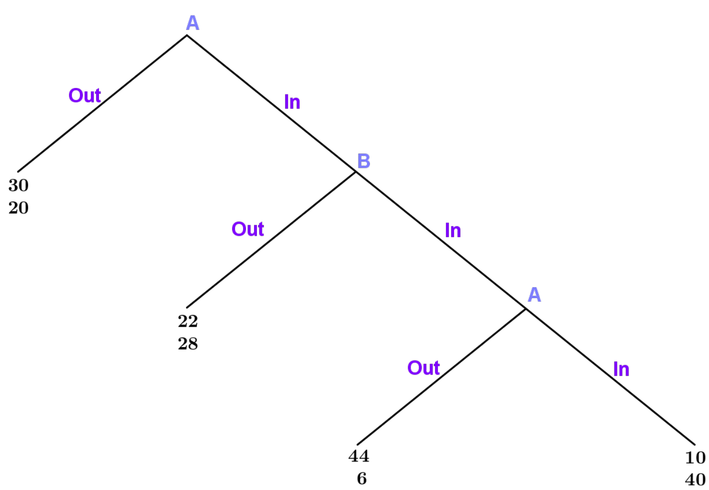


Figure 2: Constant-Sum Centipede Game

Our second treatment, the "Constant-Sum Centipede Game," is shown in Figure 2. The difference is that the sum of the players' payoffs at all stages is a constant. This version of the centipede game eliminates the *efficiency property* presented in the Baseline Centipede treatment; similar experimental treatments *without* examining players' beliefs have been conducted by Fey et.al.(1996), Levitt et.al.(2009). Further, we choose the constant-sum payoff to be 50 because this is the actual average sum of the payoffs subjects earned in the laboratory in the Baseline Centipede treatment. And we choose the split of the players' payoffs at each stage such that the cutoff probabilistic belief for each player is the same as that in the Baseline Centipede treatment. Namely, if player A expects player B to play IN with $p \geq \frac{1}{3}$, his/her best response is to play IN-OUT; if player B expects player A to play IN at the third stage with $q \geq \frac{2}{3}$, his/her best response is to play IN for the second stage.

Table 1 below summarizes the treatments and number of sessions, subjects, and matches of games for each treatment.

Table 1: Experimental Treatments

Treatments	# of Sessions	# of Subjects	Total # of Games
Baseline Centipede	5	60	450
Constant-Sum	5	60	450

Next, we list our experimental hypotheses as comparisons between the treatments, assuming that players maximize their own monetary payoffs. The first set of hypotheses describes players' rationality. As defined in Section 2, a player is rational if his or her strategy choice maximizes the expected monetary payoffs *given* his or her belief. A rational player best responds to both the initial belief and conditional belief with probability 1. In Appendix 8.1 we provide the theoretical analysis underlying these hypotheses.

Hypothesis 1. *If player A is **rational**, the frequency of A's strategy best responding to A's belief in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 2. *If player B is **rational**, the frequency of B's strategy best responding to B's belief in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Common belief of rationality implies each player's belief about their opponents' rationality at any order. Thus we further investigate players' higher order beliefs of rationality. As defined in Section 2, a player believes his or her opponent to be rational if he/she assigns probability 1 to all the states (s^{-i}, t^{-i}) in which opponent $-i$'s strategy best responds to the belief in that state. Specifically in our three-legged centipede experiment, player A's belief is the probability he/she evaluates at the root of the game tree. For player B, the probability he/she assigns to A's strategy-belief pair at the *root* of the game tree is the *initial* belief, while the probability he/she assigns once called upon to move at the second stage (if observed) is the *conditional* belief in the definition of "strong belief" in Section 2. The following hypotheses describes each player's 1st-order and 2nd-order belief of his or her opponent's rationality. In Appendix 8.1 we provide the theoretical analysis underlying these hypothesis.

Hypothesis 3. *If* ***rationality and common strong belief of rationality*** *holds, in the Baseline Centipede treatment player A believes B's rationality as frequent as he/she does in the Constant-Sum treatment.*

Hypothesis 3 comes from the fact that *rationality and common strong belief of rationality (RCSBR)* implies that player A believes in B's rationality, believes in B's (initially and conditionally) believing in A's rationality, and so on. As shown later in Section 8.1, there is no state that involves player A's believing player B will choose IN that satisfies RCSBR.

Hypothesis 4. *If* ***rationality and common strong belief of rationality*** *holds, in the Baseline Centipede treatment player B believes in A's rationality as frequently as he/she does in the Constant-Sum treatment.*

Hypothesis 5. *If* ***rationality and common initial belief of rationality*** *holds, in the Baseline Centipede treatment player B* ***initially*** *believes in A's rationality* and 2*nd-Order rationality as frequently as he/she does in the Constant-Sum treatment.*

Hypothesis 6. *If* ***rationality and common strong belief of rationality*** *holds, in the Baseline Centipede treatment player B* ***conditionally*** *believes in A's rationality* and 2*nd-Order rationality as frequently as he/she does in the Constant-Sum treatment.*

Hypothesis 4 comes from the fact that *common belief of rationality* implies that player B (both initially and conditionally) assigns probability 1 to the event of A's rationality. Hypothesis 5 comes from the fact that *rationality and common* ***initial*** *belief of rationality* implies that player B assigns probability 1 to A's rationality AND A's believing in B's rationality at the *root* of the game tree. Hypothesis 6 comes from the fact that **rationality and common *strong* belief of rationality** implies player B assigns probability 1 to A's rationality and 2nd-order rationality even after observing that A has chosen IN for the first stage.

3.2 Design and Procedure

All sessions were conducted at the Pittsburgh Experimental Economics Lab (PEEL) in Spring 2013. A total of 120 subjects are recruited from the undergraduate population of the University of Pittsburgh who have no prior experience in our experiment. The experiment adopts a between-subject design, with 5 sessions for the Baseline Centipede treatment and 5 sessions for the Constant-Sum treatment. The experiment is programmed and conducted using z-Tree (Fischbacher (2007)).

Upon arrival at the lab, we seat the subjects at separate computer terminals. After we have enough subjects to start the session[4], we hand out instructions and then read the instruction aloud. A quiz which tests the subjects' understanding of the instruction follows. We pass the quiz's answer key after the subjects finish it, explaining in private to whomever have questions.

In each session, 12 subjects participated in 15 rounds of one variation of the centipede game. Half of the subjects are randomly assigned the role of Member A and the other half the role of Member B. The role remains fixed throughout the experiment. In each round, one Member A is paired with one Member B to form a group of two. The two members in a group would then play the centipede game in that treatment. Subjects are randomly rematched with another member of the opposite role after each round.

For the aim of collecting enough data, we first used strategy method to elicit the subjects' strategy choice[5]. We asked the subjects to specify their choice at *each* decision stage had it been reached. Then the subjects' choice(s) are carried out automatically by the program and one would not have a chance to revise it if one's decision stage is reached.

After the subjects finish the choice task, they enter a "forecast task" phase which is to elicit their beliefs about their opponent's choices. Member A is asked to choose from one of the two statements which he/she thinks more likely[6]: "Member B has chosen IN" or "Member B has chosen OUT." Member A's predictions are incentivized by a linear rule: 5

[4]Each session has 12 subjects. We over-recruit as many as 16 subjects each time. By arrival time, from the 13th subject on, we pay them a \$5.00 show-up fee and ask them to leave.

[5]Another advantage of the strategy method is to exclude subjects' incentives to signal, hedge, or bluff their opponent. Had we not adopted this method, in the baseline treatment we would have observed an even higher frequency of player A's choosing IN for the first stage. Player A might find it optimal to "bluff opponent" if player B is tempted by the *efficient* and *mutually beneficial* payoff split in the Baseline Centipede treatment AND B would not strongly believe A's rationality after observing A's choosing IN for the first stage.

[6]Since in all treatments player A's cutoff probabilistic belief is $\frac{1}{3}$, which is smaller than 50 percent, the point prediction Member A is making here is without loss of generality.

points if correct, 0 if incorrect[7]. Member B is informed that his/her partner A has made a selection of choices for stage 1 and 3, AND have chosen a statement about Member B's choice. Then Member B's are asked to enter six numbers as the percent chance into a table, each cell of which represents a choice-forecast pair that Member A has chosen. For example, as shown in the table below, the upper-left cell represents the event that Member A has chosen OUT for the 1st stage and "Statement I."

Table 2: Member B's Estimation Task

Statement I	■		
Statement O			
	1st Stage *Out*, 3rd *In* or *Out*	1st Stage *In*, 3rd Stage *Out*	1st Stage *In*, 3rd Stage *In*

If B's decision stage is reached (which means his/her partner Member A has chosen IN), he/she will be asked to make a second forecast about the percent chance for each possible outcome of A's choices. Member B's predictions are incentivized by the quadratic rule:

$$5 - 2.5 \times [(1 - \beta_{ij})^2 + \sum_{kl \neq ij} \beta_{kl}^2]$$

where β_{kl} stands for Member B's stated percent chance in row k column l of the table, and i, j represents that row i column j is the outcome from Member A's choices[8].

At the end of the experiment, one round is randomly selected to count for payment. A subject's earning in each round is the sum of the points he/she earn from the choice task and the forecast task(s). The exchange rate between points and US dollars is 2.5 : 1. A subject receivers his/her earning in that selected round plus the $5.00 show-up fee.

[7]In the game that players faced with in the laboratory, player A's cutoff belief is 1/3. Thus the use of linear rule for A's belief is without loss of generality.

[8]Palfrey and Wang [27] and Wang [33] have discussed eliciting subjects' beliefs using *proper* scoring rules. This is the major reason we adopt a quadratic scoring rule. We are also aware of the risk-neutrality assumption behind the quadratic rule and the possibility to use an alternative belief elicitation method proposed by Karni [19]. But concerning the complexity of explaining Karni's method to the subjects, we adopt the quadratic rule which is simpler in explanation.

4 Experimental Findings

4.1 Rationality

In this section we present comparison results on players' rationality between the treatments. We first examine the frequency of A's best responding to his/her stated belief. Notice that there are two data points from A's strategy-belief choices that can be identified as "rational." Either player A chooses strategy IN-OUT and believes that B has chosen IN, or chooses OUT for the first stage and believes that B has chosen OUT. We sum up the frequencies from the two cases as we calculate the overall frequency of A's being rational.

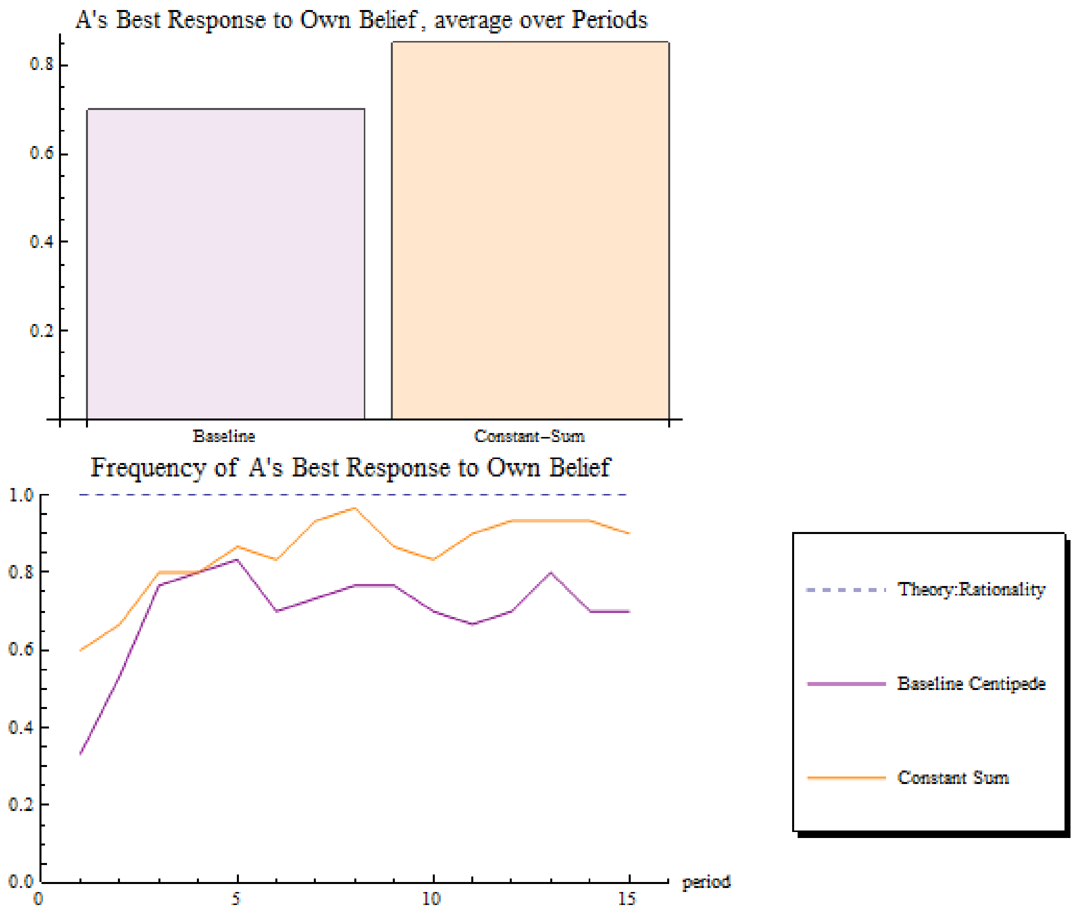
Result 1. *(1) In both treatments, the average frequency of player A being rational is below* 100 *percent. (2) The average frequency of player A being rational in the Constant-Sum treatment is significantly higher than that in the Baseline Centipede treatment.*

Result 1 addresses Hypothesis 1. Figure 3 depicts the treatment-average frequency of player A's being rational across all periods. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.0211$); but the frequency in both treatments are lower than 1 as required by the notion of *rationality.*

We then investigate the frequency of B's best responding to his/her stated belief. From B's stated belief, if the probability he/she assigns to A's choosing strategy IN-IN is greater than his/her cutoff probabilistic belief, it is rational for B to choose IN for the second stage; otherwise, it is rational to choose OUT for the second stage. We sum up the frequencies from the two cases as we calculate the overall frequency of B's being rational.

Result 2. *(1) In both treatments, the average frequency of player B being rational is below* 100 *percent. (2) The average frequency of player B being rational in the Constant-Sum treatment is significantly higher than that in the Baseline Centipede treatment.*

Result 2 addresses Hypothesis 2. Figure 4 depicts the treatment-average frequency of player B's being rational across all periods. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatment (Mann-Whitney test renders $p = 0.0946$); and both of them are significantly lower than 1 as required by the notion of *rationality.*



Note: Figure on top shows the average frequency of player A best responds to own belief over all periods all games in each treatment. Figure at bottom compares the period-wise frequency of A's best responding to own belief predicted by rationality (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

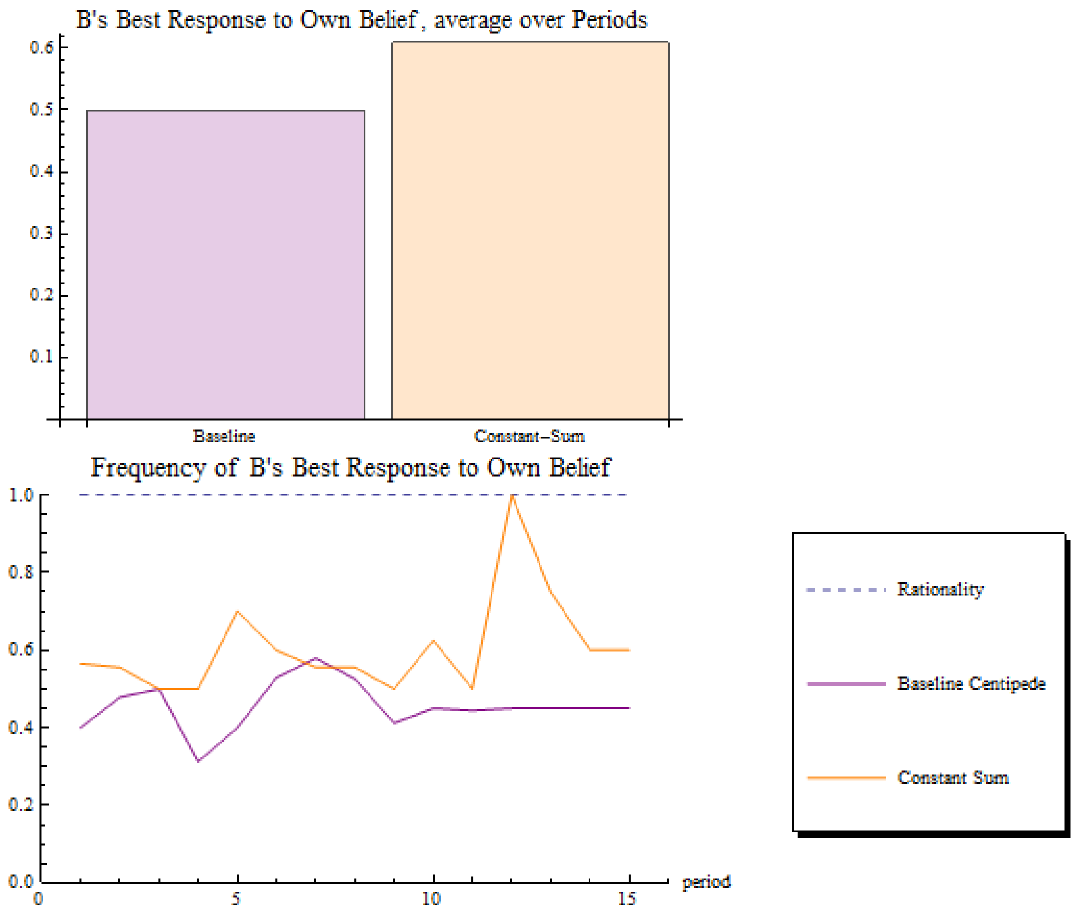
Figure 3: Average Frequency of A's Best Responding to Own Belief

4.2 Belief of Rationality and Higher-Order Belief of Rationality

In this section we present comparison results on players' belief of rationality and higher-order belief of rationality across treatments. We first examine the frequency of A believing in B's rationality conditional on A being rational herself.

Result 3. *(1) In both treatments, the average frequency of player A being rational and believing in B's rationality is below* 100 *percent. (2) The average frequency of player A being rational and believing in B's rationality in the Constant-Sum treatment is significantly higher than that in the Baseline Centipede treatment.*

Result 3 addresses Hypothesis 3. As shown in Section 3.1, if a state satisfies *rationality*

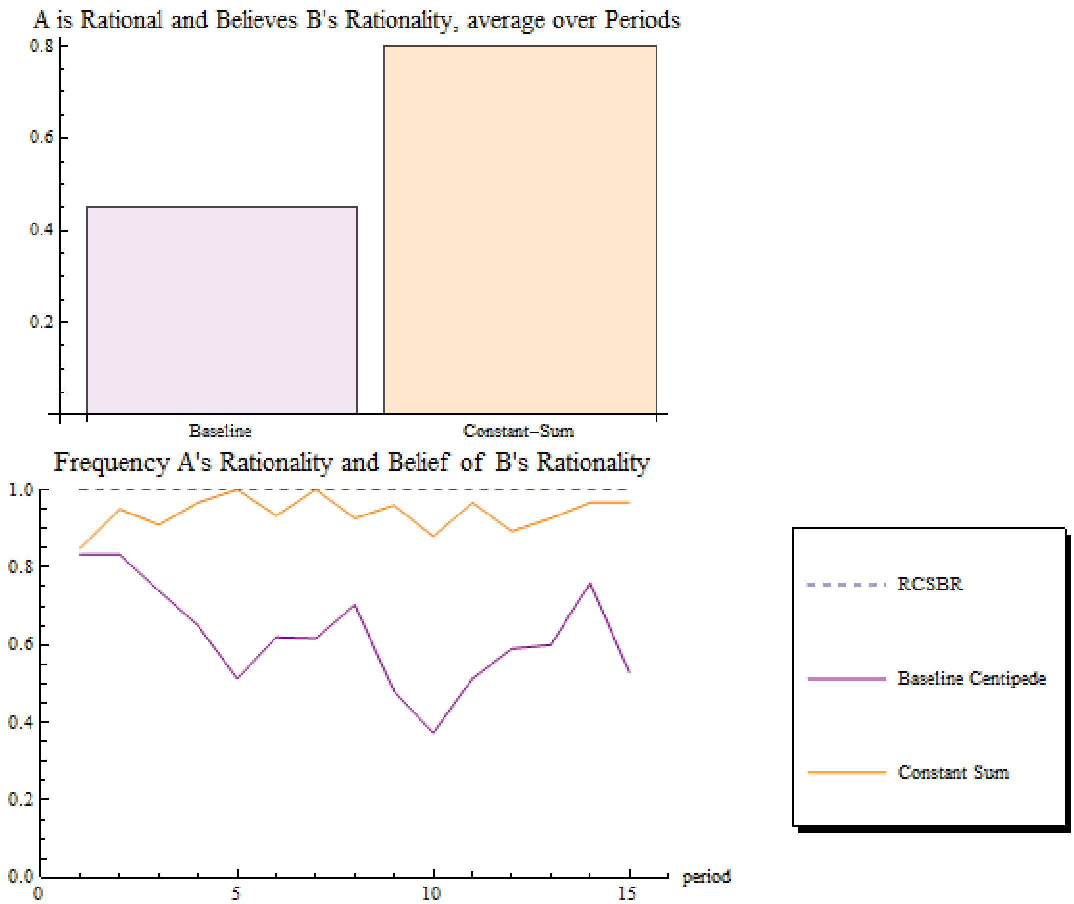


Note: Figure on top shows the average frequency of player B best responds to own belief over all periods all games in each treatment. Figure at bottom compares the period-wise frequency of B's best responding to own belief predicted by rationality (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 4: Average Frequency of B's Best Responding to Own Belief

and common strong belief of rationality, in such a state A must be rational and believe that B has chosen *Out*. Figure 5 depicts the treatment-average frequency of A being rational and believing B's rationality all periods. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.0282$); but both of them are lower than 100 percent as required by the notion of RCSBR. The comparison of player A's belief accuracy across treatments is included in Appendix 8.2.

We then examine player B's belief of A's rationality. If player B's stated belief assigns a sum of probability 1 to the two cases in which player A is rational (either A chooses strategy IN-OUT and believes B has chosen IN, or A chooses OUT for the first stage and believes B has chosen OUT), we say that player B believes A's rationality.

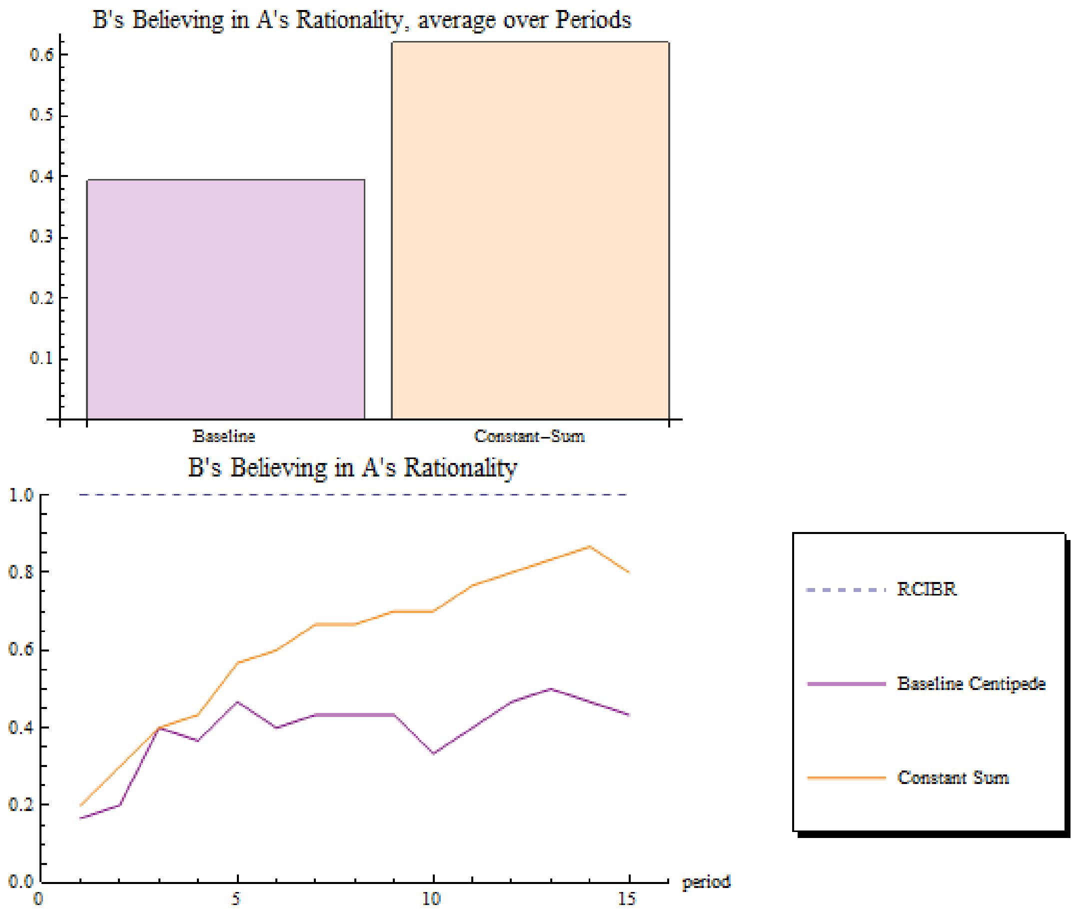


Note: Figure on top shows the average frequency of player A being rational and believing in B's rationality over all periods all games in each treatment. Figure at bottom compares the period-wise frequency of A being rational and believing in B's rationality predicted by RCSBR (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 5: Average Frequency of A's Rational and Believes in B's Rationality

Result 4. *(1) In both treatments, the average frequency of player B believing A's rationality is below* 100 *percent. (2) The average frequency of player B believing A's rationality in the Constant-Sum treatment is higher than that in the Baseline Centipede treatment.*

Result 4 addresses Hypothesis 4. Figure 6 depicts the treatment-average frequency of player B's believing A's rationality across all periods. This frequency in the Constant-Sum treatment is higher than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.1436$) ; but both of them are lower than 1 as required by the notion of *common belief of rationality.* It is also worth noting that in the Constant-Sum treatment this frequency increases towards 1 gradually as more rounds are played.



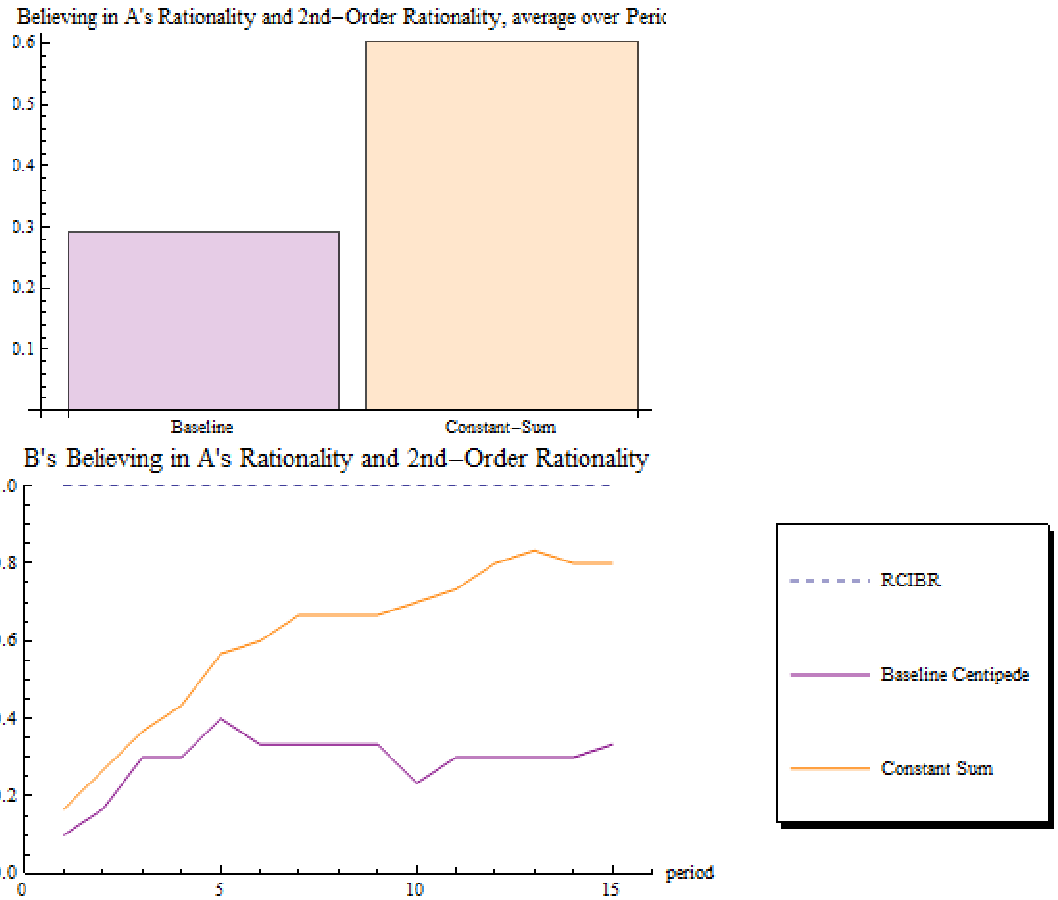
Note: Figure on top shows the average frequency of player B believing in A's rationality over all periods all games in each treatment. Figure at bottom compares the period-wise frequency of B believing in A's rationality predicted by RCIBR (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 6: B Believes in A's Rationality

Next we examine player B's believing A's rationality AND believing A's believing B's rationality (2nd-Order rationality). If player B's initial belief assigns probability 1 to the event that player A chooses OUT for the first stage and believes B has chosen OUT, we say that player B initially believes A's rationality and 2nd-Order rationality.

Result 5. *(1) In both treatments, the average frequency of player B initially believing A's rationality and 2nd-order rationality is below* 100 *percent. (2) The average frequency of player B initially believing A's rationality and 2nd-order rationality in the Constant-Sum treatment is higher than that in the Baseline Centipede treatment.*

Result 5 addresses Hypothesis 5. Figure 7 depicts the treatment-average frequency of player B believing A's rationality and 2nd-order rationality across all periods. This



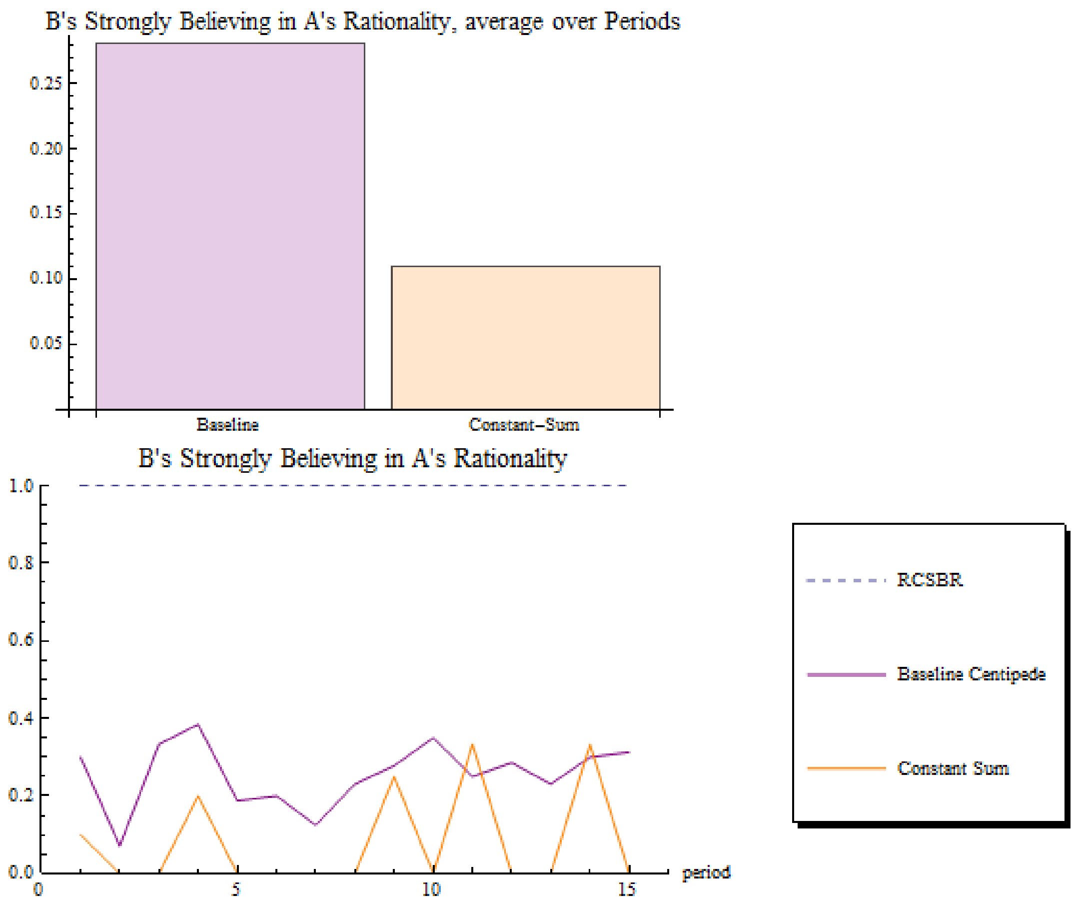
Note: Figure on top shows the average frequency of player B believing in A's rationality and 2nd-order rationality over all periods all games in each treatment. Figure at bottom compares the period-wise frequency predicted by RCIBR (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 7: B Believes in A's Rationality and 2nd-Order Rationality

frequency in the Constant-Sum treatment is higher than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.1436$); but both of them are lower than 1 as required by the notion of *rationality and common initial belief of rationality*. It is also worth noting that in the Constant-Sum treatment this frequency increases towards 1 gradually as more rounds are played.

Last we look into player B's strongly believing A's rationality AND 2nd-Order rationality conditional on A has chosen IN for the first stage. If player B's conditional belief assigns probability 1 to the event that player A chooses strategy IN-OUT and believes B has chosen IN, we say that player B strongly believes A's rationality and 2nd-Order rationality.

Result 6. *(1) In both treatments, the average frequency of player B strongly believing A's rationality and 2nd-order rationality is below* 100 *percent. (2) The average frequency of* *player B strongly believing A's rationality and 2nd-order rationality in the Constant-Sum treatment is significantly lower than that in the Baseline Centipede treatment.*



Note: Figure on top shows the average frequency of player B strongly believing in A's rationality over all periods all games in each treatment. Figure at bottom compares the period-wise frequency predicted by RCIBR (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 8: B Strongly Believing A's Rationality Conditional on Observing Unexpected Event

Result 6 addresses Hypothesis 6. Figure 8 depicts the across-period treatment-average frequency of player B's strongly believing A's rationality and 2nd-order rationality conditional on B is informed that A has chosen IN for the first stage. This frequency in the Constant-Sum treatment is lower than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.0906$); and both of them are much lower than probability 1 as required by the notion of *rationality and common strong belief of rationality.* In other words, once player B observes player A's deviating from the equilibrium path, it is even more unlikely in the Constant-Sum game for B to believe in A rationality AND A's belief in B's rationality.

5 Model: Efficiency Property and Belief Diffusion

In this section we consider a model with uncertainty in players' payoff types. We use this model to account for the belief diffusion effect in the Baseline Centipede game as shown in Section 4. The first section presents the model, assumptions, and propositions regarding the model's implication on players' strategies and beliefs. The second part explores these epistemic variables from the experimental data.

The approach we adopt here is different from the popular alternatives in accounting for experimental data, i.e. rationalizability, QRE, and the level-k models. These non-Nash-equilibrium solution concepts mainly explain anomalies in subjects' strategy choices. Since our main interest is in players' beliefs and higher-order beliefs, these non-equilibrium models lack the explanatory power. Moreover, each of these solution concepts impose additional structural restrictions. Rationalizability requires rationality and $k-$th order belief of rationality as $k \to \infty$; and when applying to experimental data it assumes that players use each possible rationalizable strategy with equal probability. Quantal response equilibrium requires one's belief being consistent with opponents' strategies; and when applying to data it imposes a logistic error structure on the probability that players fail to best respond to own beliefs. The level-k model allows a hierarchy of beliefs about opponents' rationality but assumes a $k-$th level players best respond to his or her belief about all lower level players. As shown in Section 4, these structural assumptions are not necessarily satisfied in our laboratory experiment.

The observed differences in rationality, beliefs, and higher-order beliefs of rationality in the two treatments suggest that we extend the basic epistemic analysis of Section 3. One approach is to expand the type space to include more belief types. However, as shown in Observation 1 to 5, for each player there are only a few belief types that can be part of the states that satisfy *rationality and common initial belief of rationality* and/or *rationality and common strong belief of rationality.* Including other belief types would not change the epistemic implications. Alternatively, we consider an approach that allows incomplete information regarding players' payoffs. Throughout the previous sections, we have restricted our attention to the centipede game with the monetary payoffs being the players' true payoffs. This setting implicitly assumes common knowledge of payoffs. There is no uncertainty in one's payoff, one's belief about opponent's payoff, one's belief about opponent's belief about one's own payoff, and so on. The model might better account for the data when we relax this payoff-as-common-knowledge assumption (Kneeland (2013), Weinstein and Yildiz (2011)). Notice that the only difference between the Baseline Centipede and the

Constant-Sum treatment is the sum of both players' payoffs at each stage, i.e. the efficiency property. Therefore, in the following model, we consider an additional payoff type in the Baseline centipede game: the efficiency-oriented players.

5.1 A Model with Efficiency-Oriented Players

Suppose two players face a perfect information, sequential move game with T stages. Let I denote the set of players, X^i denote the set of decision nodes for player i, H^i the collection of information set of player i, $S = S^i \times S^{-i}$ the set of (pure) strategy profiles, and Z the set of terminal nodes. Define $\pi_i : Z \to \mathbf{R}$ as player i's own payoffs. Let Π^i denote the set of player i's payoffs and define $\Pi = \Pi^i \times \Pi^{-i}$. A player's utility function is $u_i : \Pi \to \mathbf{R}$ with the general form:

$$u_i(\pi_i, \pi_{-i}) = v_i(\pi_i) + w_i(\sum_{i \in I} \pi_i)$$

with $v_i'(\cdot) > 0$ and $w_i(\cdot) \geq 0$. We assume that with probability p a player is efficiency-oriented, and this probability is commonly known to both players:

Definition 6. *A player i is **efficiency-oriented** if $w_i(\cdot) > 0$ and $w_i'(\cdot) > 0$*

Specifically, in the Baseline centipede game in Figure 1 the sum of the players' payoffs $\sum_{i \in I} \pi_i$ increases as the stage t grows, while in the Constant-Sum game in Figure 2 the sum remains constant across all periods. Without loss of generality, we re-write player i's utility at stage t as follows:

$$u_i^t(\pi_i^t, \pi_{-i}^t) = \pi_i^t + \xi_i^t, t = 1, 2, 3$$

We impose two additional assumptions on ξ_A^t and ξ_B^t, respectively:

Assumption 1. *Player A always gets higher expected utility by playing IN-OUT regardless of her belief about player B's strategy:*

$$\mu(40 + \xi_A^3) + (1 - \mu)(10 + \xi_A^2) > 20 + \xi_A^1, \forall \mu \in [0, 1]$$

Remark: First note that Assumption 1 implies that $\xi_A^{t+1} > \xi_A^t + 10$. When the game ends at the second stage, A gets higher expected utility than she does if the game ends at the first stage. Second, in the Baseline centipede game, at the third stage, the sum of the payoffs for the two terminal nodes are the same. Since $40 + \xi_A^3 > 25 + \xi_B^3$, A is always better off by choosing IN-OUT if the game reaches the third stage.

Assumption 2. *Player B always gets higher expected utility by playing IN regardless of his belief about player A's strategy:*

$$\nu(45 + \xi_B^3) + (1 - \nu)(30 + \xi_B^3) > 40 + \xi_B^2, \forall \nu \in [0, 1]$$

Remark: Note that Assumption 2 implies that $\xi_B^{t+1} > \xi_B^t + 10$. When the game ends at the third stage, B gets higher expected utility than he does if the game ends at the second stage.

Then we have the following propositions:

Proposition 1. *In the model with efficiency-oriented players and with constant-sum payoffs, the epistemic states that satisfy* ***rationality and common initial belief of rationality*** *and* ***rationality and common strong belief of rationality*** *remain unchanged as in Section 3.*

Proof: In the constant-sum centipede, the efficiency-oriented players' preference ranking over the game's outcomes is the same as the monetary-payoff maximizers: $\xi_i^t = \xi_i^{t+1}, \forall t, \forall i$. So the result follows.

Proposition 1 indicates that the type-strategy pairs of both players that satisfies RCIBR and RCSBR remains the same as in the basic model. Specifically, we have:

- A is rational if she
    - plays *Out* if she believes that B has chosen *Out*
    - plays *In-Out* if she believes that B has chosen *In*
- B is rational if he
    - plays *In* if he believes that A would play *In-In* at the third stage with probability greater than 2/3
    - plays *Out* if he believes that A would play *In-Out* at the third stage with probability less than 2/3

- A initially and strongly believes B's rationality and B's belief of A's rationality if she assigns probability 1 to B's playing *Out*

- B initially (strongly) believes A's rationality if B's conditional belief system assigns probabilities the same as the first (second) part of Observation 1

In contrast, in the baseline centipede game with the efficiency property, the epistemic states that satisfy RCIBR and RCSBR change:

Proposition 2. *In the model with efficiency-oriented players and with the sum of payoffs growing at each stage, if the players' strategies and belief types constitute a state that satisfies* ***rationality and common initial/strong belief of rationality****, then in this state:*

- *An efficiency-oriented player A plays In-Out and believes that an efficiency-oriented player B would play In*

- *An efficiency-oriented player B plays In and assigns a sum of probability* 1 *to all of A's types that play In-Out*

The proof of Proposition 2 is included in the Appendix by proving Observation 6, 7, and 7. The implications are in sharp contrast to the previous proposition about the constant-sum game: (1) an efficiency-oriented player A (B) maximizes her (his) expected payoff by playing *In-Out* (*In*), instead of playing *Out*, (2) an efficiency-oriented player expects his or her efficiency-oriented opponent to play the non-backward-induction but efficiency-improving strategy and, (3) a player who believes his or her opponent's rationality assigns positive probability to any of the opponent type that plays such a strategy.

As demonstrated in Observation 6, the model with efficiency-oriented players imposes less restrictive conditions on players' beliefs. The epistemic states that satisfy RCIBR and RCSBR involves a richer set of conditional probability systems for both players. Especially for player B, there is a continuum of belief types, each of which assigns a sum of probability one to all of player A's types that play *In-Out*. As shown in Observation 6, each of such belief type can be part of a state that satisfies RCSBR, the strong notion of common belief of rationality. Therefore, incorporating multiple payoff types relax the restriction on B's belief about A's belief: it only requires player B assigning probability 1 on all belief types of A as long as all these A's types play IN-OUT. These A's types may also includes the types do not initially or strongly believes B's rationality.

5.2 Strategies and Beliefs in the Presence of Efficiency-Oriented Players

In this section we present test hypotheses implied by the model with efficiency-oriented players and the corresponding experimental results. We first examine the implication of Proposition 1 and 2 on player A's strategy choice. In the game with the sum of payoffs growing at each stage, efficiency-oriented player A will play strategy *In-Out* regardless of her own belief about B's strategy. Thus we shall observe player A to play *In-Out* more frequently.

Hypothesis 7. *In a model with efficiency-oriented players, the observed frequency of player A choosing strategy In-Out is higher in the Baseline treatment than in the Constant-Sum treatment.*

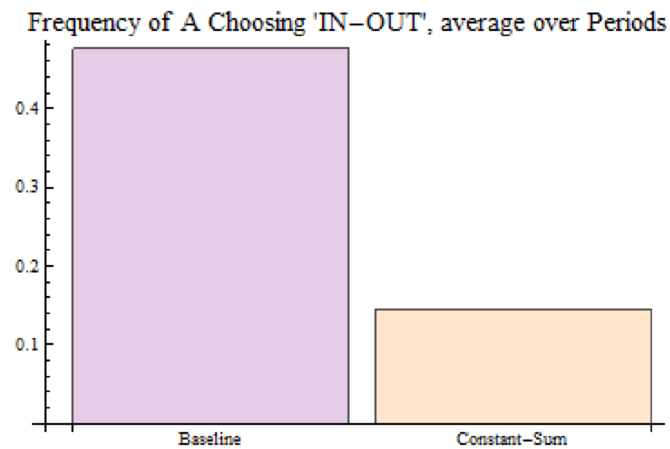
Result 7. *(1) In both treatments, the average frequency of A choosing strategy In-Out is higher than* 0*. (2) The average frequency of A choosing strategy In-Out in the Baseline treatment is significantly higher than that in the Constant-Sum treatment.*

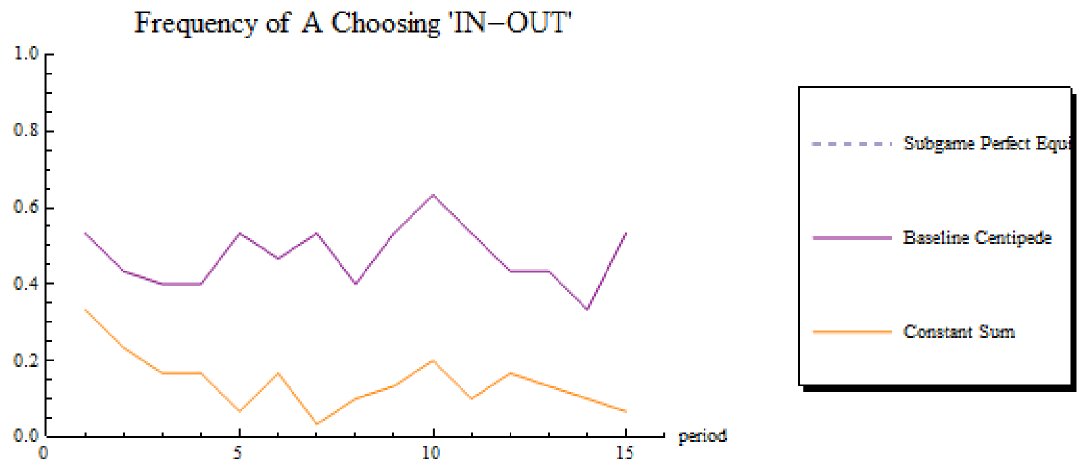
Result 7 confirms Hypothesis 7. Figure 9 depicts the treatment-average frequency of player A choosing *In-Out* across all periods. This frequency in the Baseline treatment is significantly higher than that in the Constant-Sum treatment (the Mann-Whitney test renders $p = 0.0577$); but both of them are higher than 0, as predicted by the Subgame Perfect equilibrium of the original game with only one payoff type.

Next, we investigate the implication of Proposition 1 and 2 on player A's belief about B's strategy and rationality. In the game with the sum of payoffs growing at each stage and with a portion of efficiency-oriented players, if A initially/strongly believes in B's rationality, we shall observe that A believes B to play *In* more frequently in the Baseline Centipede treatment and less frequently in the Constant-Sum treatment.

Hypothesis 8. *In a model with efficiency-oriented players, the observed frequency of player A believing B to play In is higher in the Baseline treatment than in the Constant-Sum treatment.*

Result 8. *(1) In both treatments, the average frequency of A believing B to play In is higher than* 0*. (2) The average frequency of A believing B to play In in the Baseline treatment is significantly higher than that in the Constant-Sum treatment.*





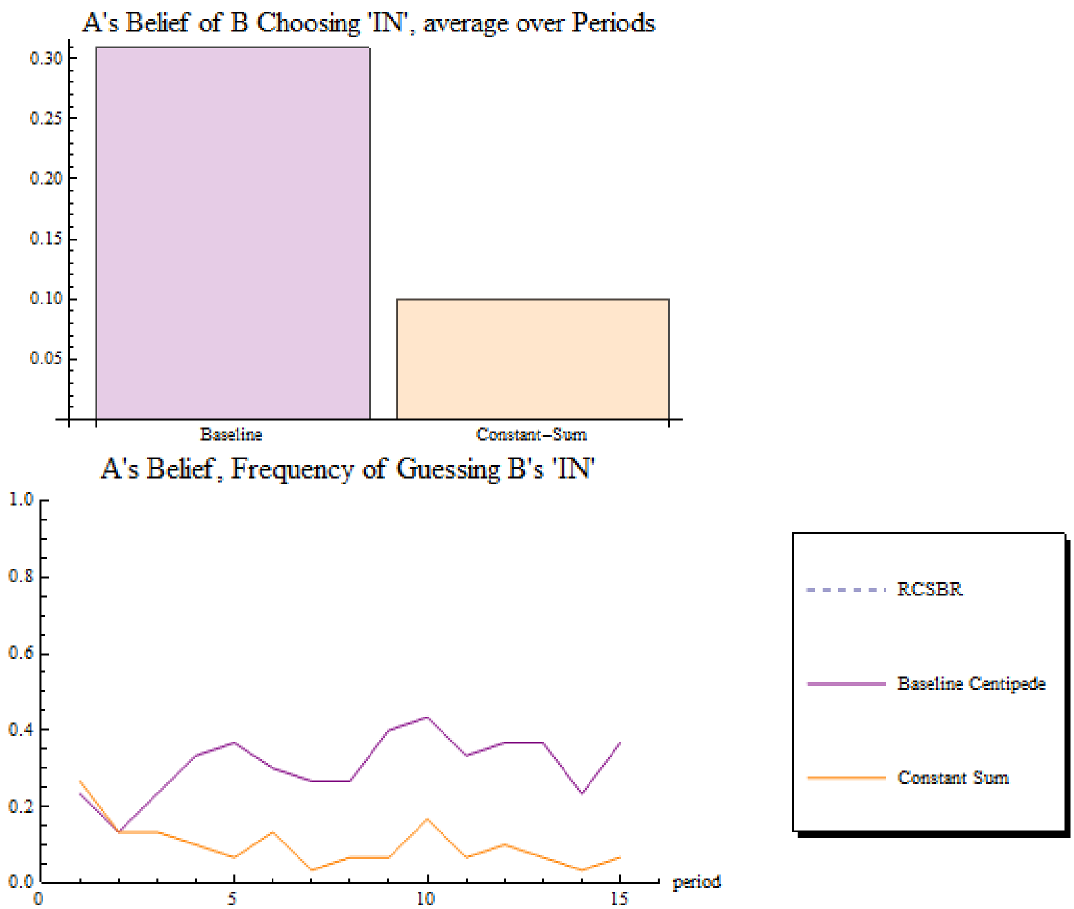
Note: Figure on top shows the average frequency of player A choosing *In-Out* over all periods all games in each treatment. Figure at bottom compares the period-wise frequency of A's choosing *In-Out* predicted by the Subgame Perfect equilibrium (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 9: Average Frequency of A's Strategy Choice

Result 8 confirms Hypothesis 8. Figure 10 depicts the treatment-average frequency of player A believing B to play *In* across all periods. This frequency in the Constant-Sum treatment is significantly lower than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.0593$); and both of them are higher than 0.

Third, we look into the implication of Proposition 1 and 2 on player B's strategy choice. In the game with the sum of payoffs growing at each stage and with a portion of the efficiency-oriented players, efficiency-oriented player B will play *In* regardless of his own belief. Thus we shall observe that B plays *In* more often in the Baseline Centipede treatment and less frequently in the Constant-Sum treatment.

Hypothesis 9. *In a model with efficiency-oriented players, the observed frequency of player B choosing In at the second stage is higher in the Baseline treatment than in the Constant-*



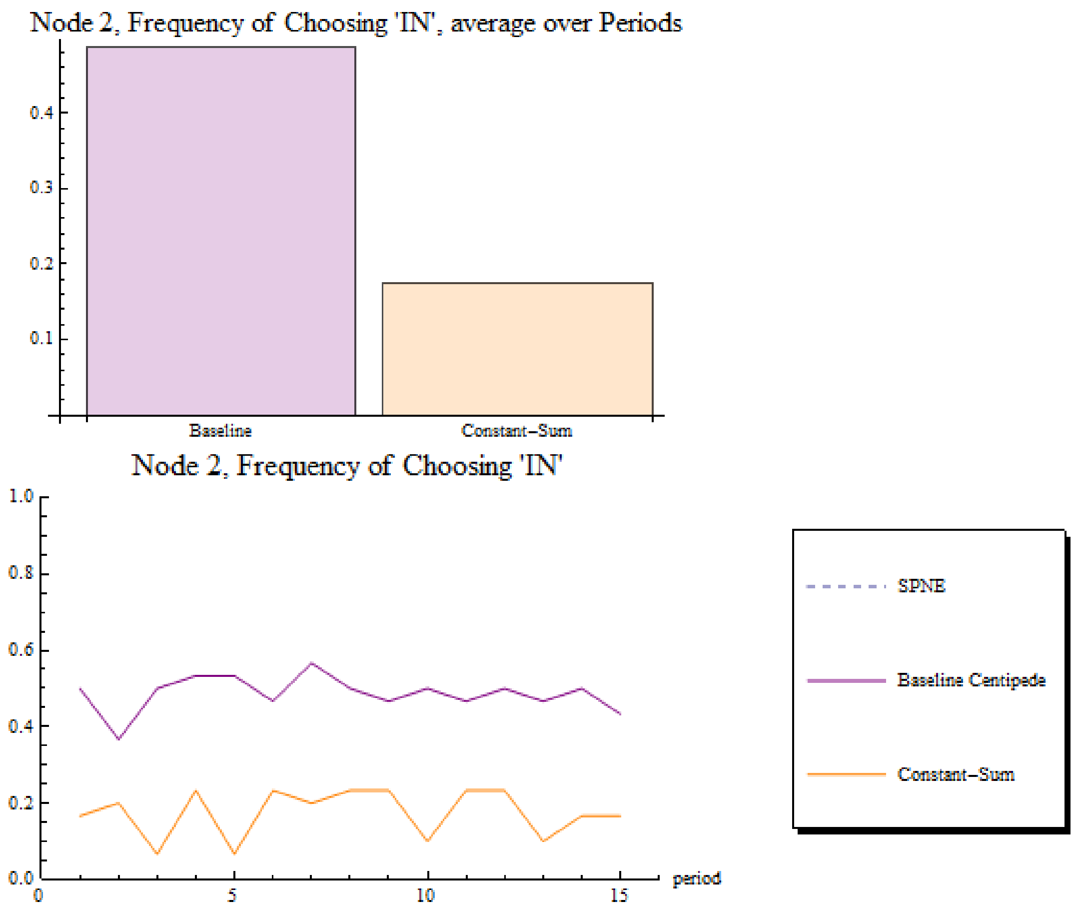
Note: Figure on top shows the average frequency of player A's belief of B choosing *In* over all periods all games in each treatment. Figure at the bottom compares the period-wise frequency of player A's belief of B's choosing IN predicted by the Subgame Perfect equilibrium (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 10: Average Frequency of A's Belief of B's Strategy and Rationality

Sum treatment.

Result 9. *(1) In both treatments, the average frequency of B choosing In at the second stage is higher than* 0*. (2) The average frequency of B choosing strategy In in the Baseline Centipede treatment is significantly higher than that in the Constant-Sum treatment.*

Result 9 confirms Hypothesis 9. Figure 11 depicts the treatment-average frequency of player B choosing strategy *In* across all periods. This frequency in the Constant-Sum treatment is significantly lower than that in the Baseline treatment (the Mann-Whitney test renders $p = 0.0367$); and both of them are higher than 0, the Subgame Perfect equilibrium prediction of the original game with only one payoff type.



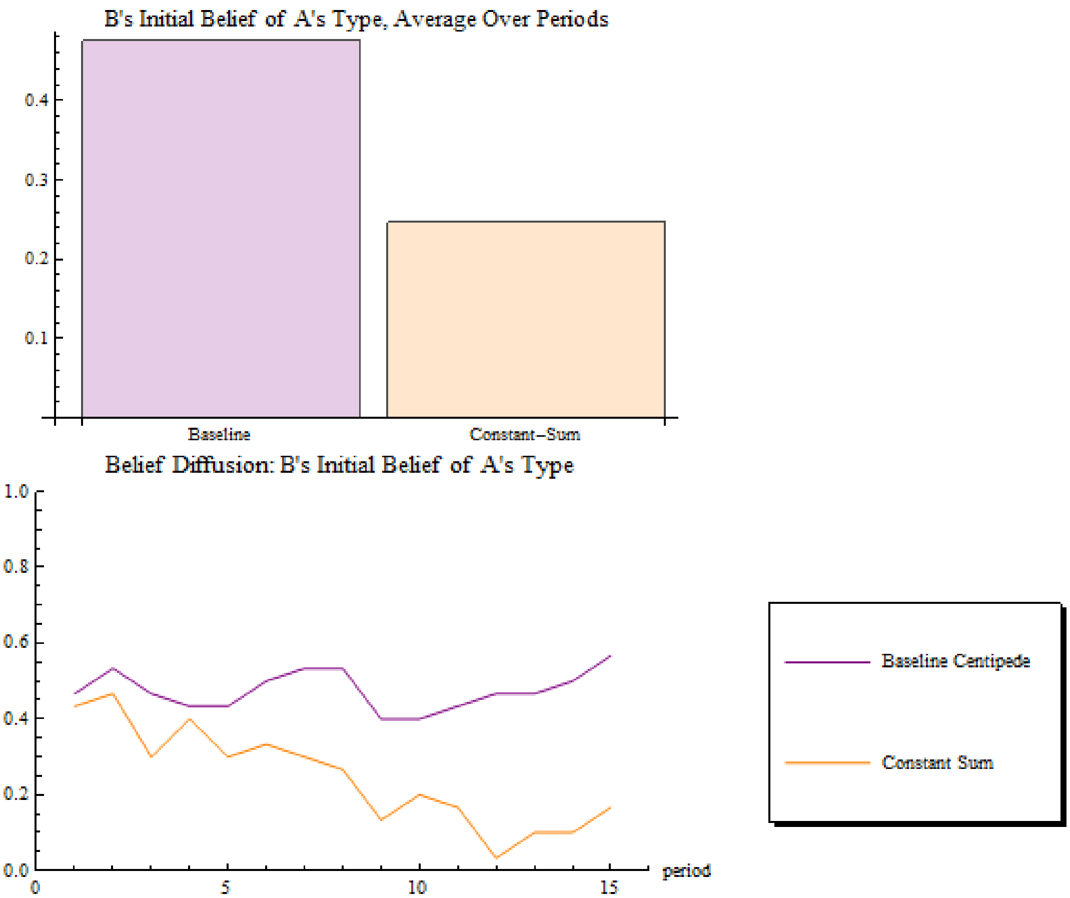
Note: Figure on top shows the average frequency of player B choosing *In* over all periods all games in each treatment. Figure at the bottom compares the period-wise frequency of B's choosing IN predicted by the Subgame Perfect equilibrium (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 11: Average Frequency of B's Strategy Choice

Finally, we examine the model's implications on the belief diffusion of player B. With efficiency-oriented players and with the sum of players' payoffs growing each stage, both RCIBR and RCISR allow player B to hold any belief about A's belief types as long as all these types of A would play strategy *In-Out*. So we shall observe that B believes A to choose *In-Out* but to believe B choosing *Out* more often in the Baseline Centipede treatment.

Hypothesis 10. *In a model with efficiency-oriented players, the observed frequency of player B holding any type of probabilistic belief about player A is higher in the Baseline treatment than that in the Constant-Sum treatment.*

Result 10. *The average frequency of B holding any type of probabilistic belief about player A in the Baseline Centipede treatment is higher than that in the Constant-Sum treatment.*



Note: Figure on top shows the average frequency of player B's belief of A choosing *In-Out* and believing that B has chosen *Out* in each treatment. Figure at the bottom compares the period-wise frequency of B's such kind of belief from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 12: Average Frequency of B's Belief of A's Belief Types

Result 10 confirms Hypothesis 10. Figure 12 depicts the treatment-average frequency of player B putting non-zero probability on A's other belief types that would also play strategy *In-Out*. This frequency is higher in the Baseline Centipede treatment and lower in the Constant-Sum treatment (the Mann-Whitney test renders $p = 0.2086$); and both of them are higher than 0, the RCIBR and RCSBR prediction of the original game with only one payoff type.

6 Related Literature

McKelvey and Palfrey's (1992) seminal centipede game experiment shows individuals' behavior inconsistent with standard game theory prediction. Neither do they find convergence to subgame perfect equilibrium prediction as subjects gain experience in later rounds of the experiment. The authors attribute such inconsistent behavior to uncertainties over players' payoff functions; specifically, the subjects might believe a certain fraction of the population is altruist. They establish a structural econometric model to incorporate players' selfish/altruistic types, error probability in actions, and error probability in beliefs. If most of the players are altruistic, the altruistic type always chooses PASS except on the last node while the selfish type might mimic the altruist for the first several moves as in standard reputation models. As pointed out, the equilibrium prediction of this incomplete information game is sensitive to the beliefs about the proportion of the altruistic type.

Subsequent experimental studies on centipede games tend to view this failure of backward induction as individuals' irrationality. Fey et.al.(1996) examine a constant-sum centipede game which excludes the possibility of Pareto improvement by not backward inducting. Among the non-equilibrium models, they find that the Quantal Response Equilibrium, in which players err when playing their best responses, fit the data best. Zauner (1999) estimates the variance of uncertainties about players' preferences and payoff types and makes comparison between the altruism models and the quantal response models. Kawagoe and Takizawa (2012) offer an alternative explanation for the deviations in centipede games adopting level-k analysis. They claim that the level-k model provide good predictions for the major features in the centipede game experiment without the complication to incorporate incomplete information on "types." Nagel and Tang (1998) investigate centipede games in a variation of the strategy method: the game is played in the reduced normal form, which is considered as "strategically equivalent" to the extensive form counterpart, but precisely to identify "learning." They examine behavior across periods according to learning direction theory. They show significant differences in patterns of choices between the cases when a player has to split the cake before her opponent and when she moves after her opponent.

Other research tries to restore the subgame perfect equilibrium outcome by providing the subjects with aids in their decision-making processes. Bornstein et. al.(2004) show that groups tend to terminate the game earlier than individual players, once free communication is allowed within each group. Maniadis (2010) examines a set of centipede games with different stakes and finds that providing aggregate information causes strong convergence to the subgame perfect equilibrium outcome. However, after uncertainties are incorporated into the payoff structure, the effect of information provision shifts in the opposite direction. Rapoport et. al.(2003) study a three-person centipede game. They show that when the number of players increases and the stakes are sufficiently high, results converge to theoretical predictions more quickly. But when the game is played with low stakes, both convergence to equilibrium and learning across iterations of the stage game are weakened. Palacios-Huerta and Volij (2009) cast their doubt on average people's full rationality by recruiting expert chess players to play a field centipede. Strong convergence to subgame perfect prediction is observed.

In this paper we elicit players' beliefs and higher-order beliefs; the underlying theoretical analysis stems from the epistemic game theory literature. To our best knowledge, there are two experiments that also intend to bring epistemology into laboratory. Both of them investigate beliefs, rationality, and beliefs of rationality in normal form games. Healy (2011) elicits subjects' preferences over outcomes, 1st-order beliefs and 2nd-order beliefs over strategies, and beliefs about opponents' rationality in a 2×2 normal form game. Following Aumann and Brandenberger (1995), Healy (2011) identifies *why* people fail to play Nash Equilibrium outcome in a normal form game. The experiment results show that subjects do not have mutual knowledge of each-others' payoffs when they fail to reach the N.E. outcome. Kneeland (2013) elicit subjects' strategic choices in a ring-network game to estimate the individual-level data on rationality, belief of rationality, and consistent beliefs about others' strategies. The laboratory data shows that no subject satisfy all three consistent epistemic conditions at the same time. The author concludes, similar to what we find in this sequential-move game, that in a normal form game failure of playing N.E. outcome might be attributed to uncertainties in players' true payoffs and players' beliefs towards these payoffs.

7 Conclusion and Discussion

This paper explores people's beliefs behind non-backward induction behavior in laboratory centipede games. We elicit the first mover's belief about the second mover's strategy as well as the second mover's initial and conditional beliefs about the first mover's strategy and 1st-order belief. The measured beliefs help to infer the conditional probability systems of both players. The inferred CPS's and players' actual strategy choices identify why they fail to reach the BI outcomes. First, we examine whether the player's strategies are best response to the stated beliefs. In both the Baseline Centipede treatment and the Constant-Sum treatment, the frequency of players' best responding to own beliefs is significantly lower than 1. Specifically, the frequency in the Constant-Sum treatment is higher than that in the Baseline treatment; and the frequency in the No-Mutual-Benefit treatment is not significantly different from that in the Baseline treatment. Second, we investigate players' belief of opponents' rationality and higher-order belief of rationality. In all treatments, both the frequency of players' believing in others' rationality and the frequency of higher-order belief of rationality are significantly smaller than 1. Nevertheless, the frequency in the Constant-Sum treatment dominates that in the Baseline and the No-Mutual-Benefit treatment. Third, when it comes to the second mover's conditional beliefs once the first-mover has chosen a non-BI strategy, the frequency of the second movers' strongly believing the first movers' rationality is very low; and there is no significant different across treatments.

8 Appendix

8.1 Proofs and Calculations

Proofs and Calculations for Section 3

This section demonstrates the hypotheses specified in the main text by proving five observations. The first observation is about B's belief of A's rationality. The rest four observations identify the states that satisfy RCIBR and RCSBE. In summary, when the strategy choices and inferred CPS's constitute a state that satisfies *rationality and common strong belief of rationality* (RCSBR henceforth), players *do not* fail to reach the backward induction (BI henceforth) outcome in this state. But the reverse is not true. It is possible that Role A's strategy choice leads to the BI outcome, but Role B's strategy and belief are not consistent with RCSBR. Moreover, there exists a state in which Role B's strategy and belief are consistent with the BI outcome but Role A's are not. There also exists a state in which neither player's strategy and belief are consistent with the BI outcome, but a weaker notion of common belief in rationality, *rationality and common initial belief of rationality*, still holds.

For the east of demonstration, we alter the notations of the players' moves slightly, as shown in Figure 13. Since we shall prove the following observations for all three treatments, we use (x_j, y_j) to represent the players' payoffs associated with each terminal node. And u^A represents Statement OUT, t^A represents Statement IN in the instruction.

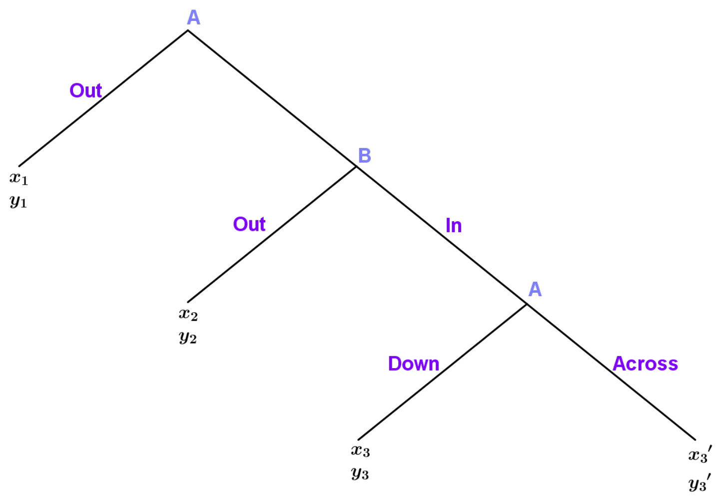


Figure 13: The Centipede Game

Observation 1. *From the measured initial belief of player B, if* $\beta_{11} + \beta_{22} = 1$*, player B*

Table 3: Measured Initial Belief of Player B

	Out	Down	Across
u^A	β_{11}	β_{12}	β_{13}
t^A	β_{21}	β_{22}	β_{23}

initially believes player A's rationality.

From the measured conditional belief of player B, if $\gamma_{22} = 1$*, player B strongly believes*

Table 4: Measured Conditional Belief of Player B

	Down	Across
u^A	γ_{12}	γ_{13}
t^A	γ_{22}	γ_{23}

player A's rationality and 2nd-Order rationality.

Observation 2. If the following data point is observed, the players' strategies and beliefs constitute a state that satisfies RCSBR:

- Role A chooses *Out* and statement u^A

- Role B chooses *Out* and the measured beliefs take the form:

Table 5: RCSBR Beliefs

u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[1]	0[0]
	Out	Down	Across

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Observation 3. If the following data point is observed, Role B's strategy and belief are not consistent with RCSBR. Nevertheless, the BI outcome still obtains.

- Role A chooses *Out* and statement u^A

- Role B chooses *In* and the measured beliefs take the form:

Table 6: Backward Induction without RCSBR Beliefs

u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[0]	0[1]
	Out	Down	Across

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Remark: *1st-order strong belief of rationality* of both players because Role B's measured belief indicates that he does not *strongly* believe Role A's rationality. However, since Role A chooses *Out* at the first node, the BI outcome still obtains. Although RCSBR does not hold in this state, a weaker notion, *rationality and common initial belief of rationality* (RCIBR), still holds. RCIBR only requires the belief consistency given the root of the game tree.

Observation 4. If the following data point is observed, Role B's strategy and belief are consistent with the BI outcome. But the BI outcome does not obtain.

- Role A chooses *Down* and statement t^A

- Role B chooses *Out* and the measured beliefs take the form:

Table 7: Player B's Strong Belief of Rationality in Non-BI Outcome

	Out	Down	Across
u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[1]	0[0]

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Remark: In this state there is no *1st-order **strong** belief of rationality*, nor *1st-order **initial** belief of rationality* because Role A's measured belief indicates that she does not strongly, nor initially believe Role B's rationality. Since Role A chooses *Down* and Role B chooses *Out*, the BI outcome does not obtain. The game ends at the second node by Role B's playing *Out*.

Observation 5. If the following data point is observed, neither player's strategy and belief is consistent with the BI outcome. The BI outcome does not obtain. Nevertheless, the strategies and beliefs constitute a state that satisfies *rationality and common **initial** belief of rationality.*

- Role A chooses *Down* and statement t^A
- Role B chooses *In* and the measured beliefs take the form:

Table 8: No RCSBR and Non-Backward-Induction Outcome

	Out	Down	Across
u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[0]	0[1]

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Remark: In this state there is no *1st-order strong belief of rationality* of both players because (1) Role B's measured belief indicates that he does not *strongly* believe Role A's rationality, and (2) Role A's measured beliefs indicates that she believes Role B's rationality in response to his belief, but she does not believe that Role B believes her rationality. Since Role A chooses *Down* and Role B chooses *In*, the BI outcome does not obtain. The game ends at the last node by Role A's choosing *Down*.

Compare observation 3 and 5. Role B's inferred CPS is the same, which assigns probability 0 to Role A's rationality if Role B's decision node is reached. Therefore, whenever Role B does not believe Role A's rationality conditional on a zero-probability event, Role A can attain a higher payoff by playing *Down* instead of *Out*. Both states satisfy *rationality and common* ***initial*** *belief of rationality*, but not *rationality and common* ***strong*** *belief of rationality*.

8.1.1 Proof for Observation 2 and 4

The inferred CPS's of both players are as follows:

Table 9: Proof for Observation 7 and 9

$\lambda^a(t^a)$

T^b		Out	In
	t^b	0	1

S^b

$\lambda^a(u^a)$

T^b		Out	In
	t^b	1[0]	0[1]

S^b

$\lambda^b(t^b)$

T^a		Out	Down	Across
	u^a	1[0]	0[0]	0[0]
	t^a	0[0]	0[1]	0[0]

S^a

We are going to show:

1. The state $(\text{Out}, u^a, \text{Out}, t^b)$ satisfies both RCIBR and RCSBR
2. The state $(\text{Down}, t^a, \text{Out}, t^b)$ satisfies neither RCIBR nor RCSBR

First notice that the strategy-type pair (Out, u^a) and (Down, t^a) are rational for player Ann. The strategy-type pair (Out, t^b) is rational for player Bob. For the **initial** belief we have:

$$\mathrm{IB}^a(R_1^b) = \{u^a\}, \mathrm{IB}^b(R_1^a) = \{t^b\},$$

then we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \mathrm{IB}^a(R_1^b)) = \{(\mathrm{Out}, u^a)\} \\ R_2^b &= R_1^b \cap (S^b \times \mathrm{IB}^b(R_1^a)) = \{(\mathrm{Out}, t^b)\} \end{aligned}$$

Inductively, we have $R_m^a\{(\mathrm{Out}, u^a)\}$ and $R_m^b = (\mathrm{Out}, t^b)$, $\forall m \in \mathbf{N}$. Therefore we have:

$$\cap_{m=1}^{\infty} R_m = \{(\mathrm{Out}, u^a, \mathrm{Out}, t^b)\}$$
$$\text{and } (\mathrm{Down}, t^a, \mathrm{Out}, t^b) \notin \cap_{m=1}^{\infty} R_m$$

As for **strong** beliefs, at the second node of the game, Bob's information set $H = \{\text{Ann would play "Down" or "Across"}\}$. Thus

$$H \times T^a = \{(\mathrm{Down}, t^a), (\mathrm{Down}, u^a), (\mathrm{Across}, t^a), (\mathrm{Across}, u^a)\}$$

Bob's type t^b is the only type who assigns probability 1 to any event E s.t. $E \cap (H \times T^a) \neq \emptyset$. So we have $\mathrm{SB}^b(R_1^a) = \{t^b\}$.

At the first node of the game, $H = \{\text{Bob would play "Out" or "In"}\}$ for Ann. So Ann's strong beliefs at this node is the same as her initial belief. At the third node of the game, Ann's information set $H = \{\text{Bob played "In"}\}$. Both Ann's type assigns probability 1 to any event E s.t. $E \cap (H \times T^a) \neq \emptyset$. So we have $\mathrm{SB}^a(R_1^b) = \{t^a, u^a\}$.

Inductively we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \mathrm{SB}^a(R_1^b)) = R_1^a \\ R_2^b &= R_1^b \cap (S^b \times \mathrm{SB}^b(R_1^a)) = \{(\mathrm{Out}, t^b)\} \end{aligned}$$

Iterate one more level, we have:

$$\begin{aligned} \mathrm{SB}^b(R_2^a) &= \mathrm{SB}^b(R_1^a) = \{t^b\} \\ \mathrm{SB}^a(R_2^b) &= \{t^a \in T^a : \forall H \text{ s.t. } R_2^b \cap (H \times T^b) \neq \emptyset, \lambda^a(t^a)(R_2^b) = 1\} \\ &= \{u^a\} \end{aligned}$$

and

$$R_3^a = R_2^a \cap (S^a \times \mathrm{SB}^a(R_2^b)) = \{(\mathrm{Out}, u^a)\}$$
$$R_3^b = R_2^b \cap (S^b \times \mathrm{SB}^b(R_2^a)) = \{(\mathrm{Out}, t^b)\}$$

Then we have $\mathrm{SB}^b(R_3^a) = \{t^b\}$ and $\mathrm{SB}^a(R_3^b) = \{u^a\}$. For any $m \geq 3$, we have $R_m^a = \{(\mathrm{Out}, u^a)\}$ and $R_m^b = \{(\mathrm{Out}, t^b)\}$. Thus:

$$\cap_{m=1}^{\infty} R_m = \{(\mathrm{Out}, u^a, \mathrm{Out}, t^b)\}$$

Therefore, the only state that satisfies *rationality and common strong belief of rationality* is $(\mathrm{Out}, u^a, \mathrm{Out}, t^b)$.

The results are summarized in the following table:

Table 10: Summary for Proof for Observation 7 and 9

State	RCIBR	RCSBR
$(\mathrm{Down}, t^a, \mathrm{Out}, t^b)$	×	×
$(\mathrm{Out}, u^a, \mathrm{Out}, t^b)$	√	√

8.1.2 Proof for Observation 3 and 5

The inferred CPS's of both players are:

We are going to show:

- Both states $(\mathrm{Down}, t^a, \mathrm{In}, t^b)$ and $(\mathrm{Out}, u^a, \mathrm{In}, t^b)$ satisfy RCIBR but not RCSBR.

We first identify strategy-type pairs that are rational. For player Ann, it is easy to show that strategy $s^a = \mathrm{Down}$ maximizes type t^a's expected payoff and strategy $s^a = \mathrm{Out}$ maximizes type u^a's expected payoff. For player Bob, strategy $s^b = \mathrm{In}$ maximizes type t^b's expected payoff. Thus we have:

$$R_1^a = \{(\mathrm{Down}, t^a), (\mathrm{Out}, u^a)\}$$
$$R_1^b = \{(\mathrm{In}, t^b)\}$$

Table 11: Proof for Observation 8 and 10

$\lambda^a(t^a)$

T^b	S^b: Out	In
t^b	0	1

$\lambda^a(u^a)$

T^b	S^b: Out	In
t^b	1[0]	0[1]

$\lambda^b(t^b)$

T^a	S^a: Out	Down	Across
u^a	1[0]	0[0]	0[0]
t^a	0[0]	0[0]	0[1]

Initial beliefs:

Both Ann's type t^a and u^aassign probability 1 to (In, t^b), so we have $\text{IB}^a(R_1^b) = \{t^a, u^a\}$. Bob's type t^b assigns probability 1 to $(\text{Out}, u^a) \in R_1^a$, so we have $\text{IB}^b(R_1^a) = \{t^b\}$.

Then we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \text{IB}^a(R_1^b)) \\ &= \{(\text{Down}, t^a), (\text{Out}, u^a)\} \cap (\{\text{Out}, \text{Down}, \text{Across}\} \times \{t^a, u^a\}) \\ &= R_1^a \end{aligned}$$

Similarly, $R_2^b = R_1^b$, and $R_3^a = R_2^a \cap (S^a \times \text{IB}^a(R_2^b)) = R_2^a = R_1^a$. Mathematical induction gives:

$$R_m^a = R_{m-1}^a = R_1^a \Rightarrow R_{m+1}^a = R_m^a = R_1^a$$

Similar result for Bob. Therefore we have

$$\begin{aligned} R_m &= R_m^a \times R_m^b = R_{m-1} \\ &\Rightarrow \cap_{m=1}^{\infty} R_m = R_1^a \times R_1^b \\ &= \{(\text{Down}, t^a, \text{In}, t^b), (\text{Out}, u^a, \text{In}, t^b)\} \end{aligned}$$

Thus both states satisfy *rationality and common initial belief of rationality.*

Strong beliefs:

At the second node of the game, Bob's information set $H = \{\text{Ann would play “Down” or “Across”}\}$.

Thus

$$H \times T^a = \{(\text{Down}, t^a), (\text{Down}, u^a), (\text{Across}, t^a), (\text{Across}, u^a)\}$$

Bob's type t^b assigns probability 0 to $(\text{Down}, t^a) \in R_1^a$, but assigns probability 1 to $(\text{Across}, t^a) \notin R_1^a$. So we have $\text{SB}^b(R_1^a) = \emptyset$. Thus $\cap_{m=1}^{\infty} R_m = \emptyset$. No state belongs to $\emptyset$. Hence neither state satisfies RCSBR.

The results are summarized in the following table:

Table 12: Summary of Proof for Observation 8 and 10

State	RCIBR	RCSBR
$(\text{Down}, t^a, \text{In}, t^b)$	$\surd$	$\times$
$(\text{Out}, u^a, \text{In}, t^b)$	$\surd$	$\times$

Proofs for Section 5

This section derives epistemic implications of the model in Section 5. First, we observe that the model with efficiency-oriented players allows a richer set of B's belief types. Specifically, RCIBR and RCSBR imposes less restrictive conditions on B's belief about A's belief types. Second, we show that A's playing DOWN, A's believing B's playing IN, and B's playing IN with a continuum of belief types satisfy both RCIBR and RCSBR.

Observation 6. *From the measured initial belief of player B, player B initially believes*

Table 13: Measured Initial Belief of Player B, with Efficiency-Oriented Players

	Out	Down	Across
u^A	β'_{11}	β'_{12}	β'_{13}
t^A	β'_{21}	β'_{22}	β'_{23}

player A's rationality if and only if the $\beta'_{\{\cdot\}}$ *satisfy*

- $\beta'_{11} = \beta'_{21} = \beta'_{13} = \beta'_{23} = 0$

- $\beta'_{12} = x, \beta'_{22} = 1 - x, \forall x \in [0, 1]$

From the measured conditional belief of player B, player B strongly believes player A's rationality and 2nd-Order rationality if and only if the $\gamma'_{\{\cdot\}}$ *satisfy*

Table 14: Measured Conditional Belief of Player B, with Efficiency-Oriented Players

u^A	γ'_{12}	γ'_{13}
t^A	γ'_{22}	γ'_{23}
	Down	Across

- $\gamma'_{13} = \gamma'_{23} = 0$
- $\gamma'_{12} = y, \gamma'_{22} = 1 - y, \forall y \in [0, 1]$

Proof: As defined in Section 5, player A maximizes her expected utility by playing DOWN regardless of her belief about player B. So any type of player B who believes in A's rationality should assign probability 1 to A's playing DOWN regardless of A's types and assign probability 0 to any other strategy-type pairs of player A. Moreover, since any type of player A maximizes utility by choosing DOWN, a B's type who initially (strongly) believes in A's rationality can assign any number $x(y)$ in $[0, 1]$ to A's strategy-type pair $(t^A, \text{Down}), \forall t^A \in T^A$.

Observation 7. If the following data point is observed, then in the model with efficiency-oriented players, the players' strategies and beliefs constitute a state that satisfies both RCIBR and RCSBR:

- *Role A chooses Down and statement* t^A
- *Role B chooses In and the measured beliefs take the form:*

u^A	*0[0]*	$x[y]$	*0[0]*
t^A	*0[0]*	$1 - x[1 - y]$	*0[0]*
	Out	*Down*	*Across*

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Observation 8. If the following data point is observed, then in the model with efficiency-oriented players, neither RCIBR nor RCSBR is satisfied:

- *Role A chooses Down and statement* u^A
- *Role B chooses In and the measured beliefs take the form:*

u^A	*0[0]*	x*[*y*]*	*0[0]*
t^A	*0[0]*	$1-x$*[*$1-y$*]*	*0[0]*
	Out	*Down*	*Across*

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

8.1.3 Proof for Observation 7 and 8

The inferred CPS's of both players are as follows:

$\lambda^a(t^a)$

T^b	t^b	0	1
		Out	In
		S^b	

$\lambda^a(u^a)$

T^b	t^b	1[0]	0[1]
		Out	In
		S^b	

$\lambda^b(t^b)$

T^a	u^a	$x[y]$	0[0]	0[0]
	t^a	0[0]	$1-x[1-y]$	0[0]
		Out	Down	Across
			S^a	

We are going to show:

1. The state $(\text{Down}, t^a, \text{In}, t^b)$ satisfies both RCIBR and RCSBR
2. The state $(\text{Down}, u^a, \text{In}, t^b)$ satisfies neither RCIBR nor RCSBR

First notice that the strategy-type pair (Down, t^a) and (Down, u^a) are rational for player Ann. The strategy-type pair (In, t^b) is rational for player Bob. For the **initial** belief we have:

$$\text{IB}^a(R_1^b) = \{t^a\}, \text{IB}^b(R_1^a) = \{t^b\},$$

then we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \mathrm{IB}^a(R_1^b)) = \{(\mathrm{Down}, t^a)\} \\ R_2^b &= R_1^b \cap (S^b \times \mathrm{IB}^b(R_1^a)) = \{(\mathrm{In}, t^b)\} \end{aligned}$$

Inductively, we have $R_m^a\{(\mathrm{Down}, t^a)\}$ and $R_m^b = (\mathrm{In}, t^b)$, $\forall m \in \mathbf{N}$. Therefore we have:

$$\cap_{m=1}^{\infty} R_m = \{(\mathrm{Down}, t^a, \mathrm{In}, t^b)\}$$
$$\text{and } (\mathrm{Down}, u^a, \mathrm{In}, t^b) \notin \cap_{m=1}^{\infty} R_m$$

As for **strong** beliefs, at the second node of the game, Bob's information set $H = \{$Ann would play "Down" or "Across"$\}$. Thus

$$H \times T^a = \{(\mathrm{Down}, t^a), (\mathrm{Down}, u^a), (\mathrm{Across}, t^a), (\mathrm{Across}, u^a)\}$$

Bob's type t^b is the only type who assigns probability 1 to any event E s.t. $E \cap (H \times T^a) \neq \emptyset$. So we have $\mathrm{SB}^b(R_1^a) = \{t^b\}$.

At the first node of the game, $H = \{$Bob would play "Out" or "In"$\}$ for Ann. So Ann's strong beliefs at this node is the same as her initial belief. At the third node of the game, Ann's information set $H = \{$Bob played "In"$\}$. Both Ann's type assigns probability 1 to any event E s.t. $E \cap (H \times T^a) \neq \emptyset$. So we have $\mathrm{SB}^a(R_1^b) = \{t^a\}$.

Inductively we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \mathrm{SB}^a(R_1^b)) = \{(\mathrm{Down}, t^a)\} \\ R_2^b &= R_1^b \cap (S^b \times \mathrm{SB}^b(R_1^a)) = \{(\mathrm{In}, t^b)\} \end{aligned}$$

Iterate one more level, we have:

$$\begin{aligned} \mathrm{SB}^b(R_2^a) &= \mathrm{SB}^b(R_1^a) = \{t^b\} \\ \mathrm{SB}^a(R_2^b) &= \{t^a \in T^a : \forall H \text{ s.t. } R_2^b \cap (H \times T^b) \neq \emptyset, \lambda^a(t^a)(R_2^b) = 1\} \\ &= \{t^a\} \end{aligned}$$

and

$$R_3^a = R_2^a \cap (S^a \times \mathrm{SB}^a(R_2^b)) = \{(\mathrm{Down}, t^a)\}$$
$$R_3^b = R_2^b \cap (S^b \times \mathrm{SB}^b(R_2^a)) = \{(\mathrm{In}, t^b)\}$$

Then we have $\mathrm{SB}^b(R_3^a) = \{t^b\}$ and $\mathrm{SB}^a(R_3^b) = \{t^a\}$. For any $m \geq 3$, we have $R_m^a = \{(\mathrm{Down}, t^a)\}$ and $R_m^b = \{(\mathrm{In}, t^b)\}$. Thus:

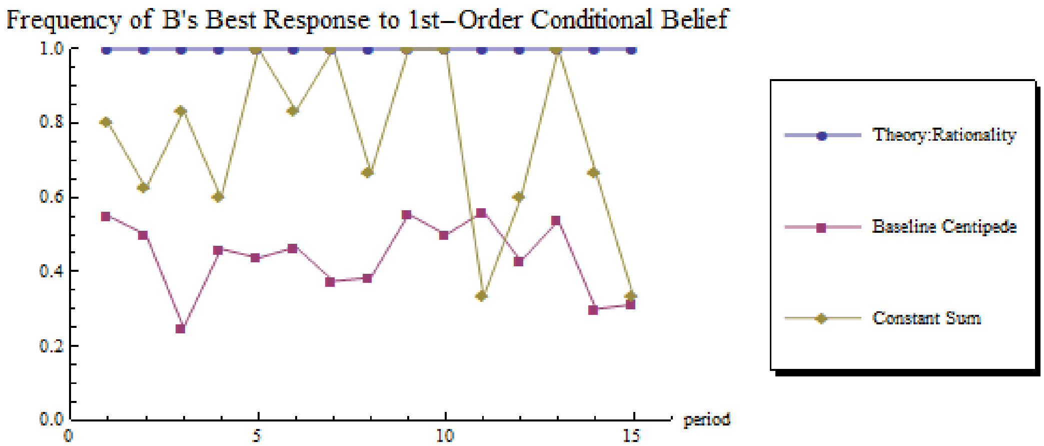
$$\cap_{m=1}^{\infty} R_m = \{(\mathrm{Down}, t^a, \mathrm{In}, t^b)\}$$

Therefore, the only state that satisfies *rationality and common strong belief of rationality* is $(\mathrm{Down}, t^a, \mathrm{In}, t^b)$.

The results are summarized in the following table:

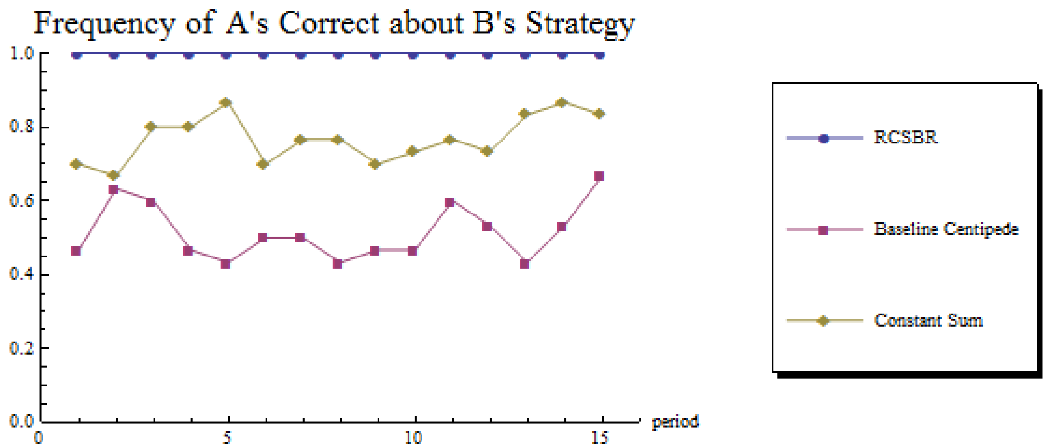
State	RCIBR	RCSBR
$(\mathrm{Down}, t^a, \mathrm{In}, t^b)$	$\surd$	$\surd$
$(\mathrm{Down}, u^a, \mathrm{In}, t^b)$	$\times$	$\times$

8.2 Other Figures and Tables



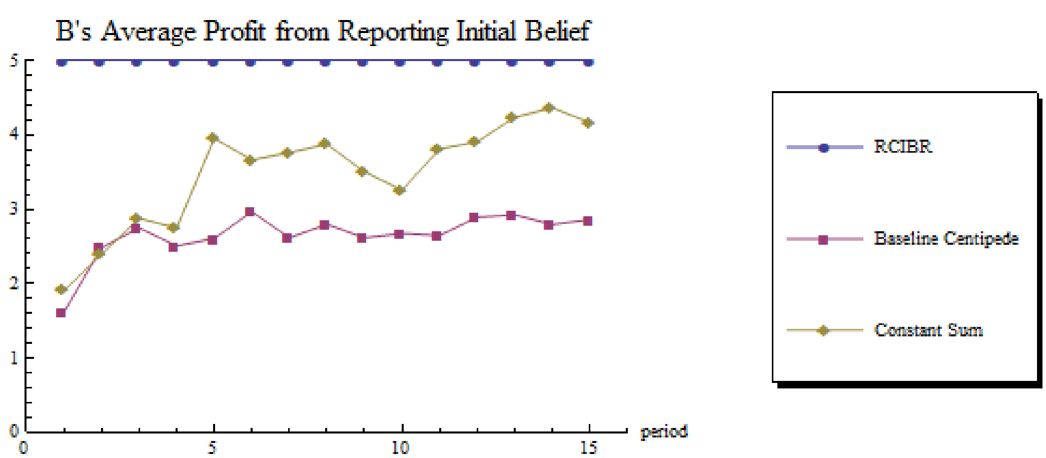
Note: Figure **??** compares the average frequency of B's best responding to his/her stated conditional belief if B is conditionally consistent (blue curve) versus the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve).

Figure 14: Average Frequency of B's Best Responding to Own Conditional Belief, Across Periods



Note: Figure on top compares the average accuracy of A's belief if RCBR holds (blue curve), the actual accuracy from the Baseline Centipede treatment (purple curve), versus the actual accuracy from the Constant-Sum treatment (yellow curve).

Figure 15: Accuracy of A's Belief, Across Periods



Note: Figure on top compares the average accuracy of B's initial belief if RCIBR holds(blue curve), versus the actual accuracy from the Baseline Centipede treatment (purple curve), and the actual accuracy from the Constant-Sum treatment (yellow curve).

Figure 16: Accuracy of B's Belief, Across Periods

8.3 Laboratory Instructions

INSTRUCTIONS

Welcome! Thank you for participating in this experiment. This experiment studies decision-making between two individuals. In the following one hour or less, you will participate in 15 rounds of decision making. Please read the instructions carefully; the cash payment you earn at the end of the experiment may depend on how well you understand the instructions and make your decisions accordingly.

Your Role and Decision Group

Half of the participants will be randomly assigned the role of Member A and half will be assigned the role of Member B. Your role will remain fixed throughout the experiment. In each round, one Member A will be paired with one Member B to form a group of two. The two members in a group make decisions that will affect their earnings in the round. Participants will be randomly rematched with another member of the opposite role after each round.

Your Choice Task(s) in Each Round

In each round, each group will face the three-stage decision task shown in Figure **??**. The nodes represent choice stages, the letters above the nodes represent the member who is going to make a choice, and the numbers represent the points one will earn, with A's points on top and B's points at bottom.

- In the 1st stage A must decide between two options: *Out* or *In*. If A chooses *Out*, the task ends with A receiving 20 and B 10 points. If A chooses *In*, the task proceeds to the 2nd stage.
- In the 2nd stage B must decide between two options: *Out* or *In*. If B chooses *Out*, the task ends with A receiving 10 and B 40 points. If B chooses *In*, the task proceeds to the 3rd stage.
- In the 3rd stage A must choose again between two options: *Out* or *In*. If A chooses *Out*, A will receive 40 and B 30 points. If A chooses *In*, A will receive 25 and B 45 points.

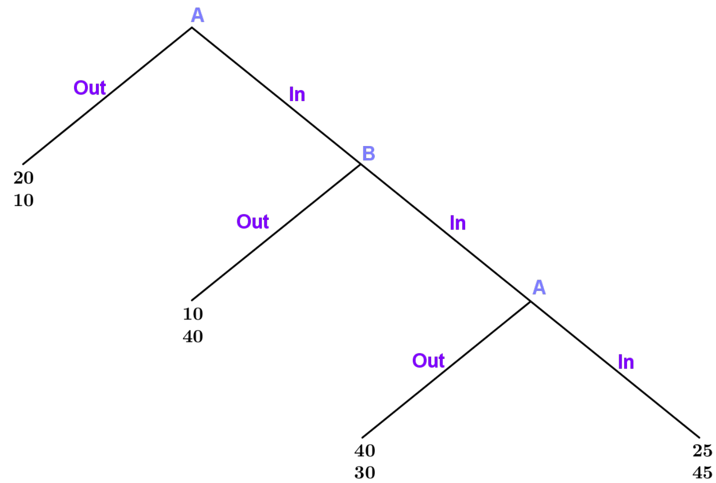


Figure 17: The Decision Task

<u>Member A's Choice Task</u>

You will be asked to specify your choices for **both** stage 1 and 3 through a computer interface. For each stage, you can choose one and only one option. Note that you will be making your choices **at the same time** your partner B is making his or her choice. So you don't know what B chooses. The choices you make here will be carried out automatically by the computer later on. You will not have an opportunity to revise them.

<u>Member B's Choice Task</u>

You will be asked to specify your choice for stage 2 through a computer interface. You can choose one and only one option. Note that you will be making your choice **at the same time** your partner A is making his or her choices. So you don't know what A chooses. The choice you make here will be carried out automatically by the computer later on. You will not have an opportunity to revise it.

<u>Forecast Tasks in Each Round</u>

Besides having the opportunity to earn points in the choice task, you will also be given the opportunity to earn extra points by making forecast(s).

<u>Member A's Forecast Task</u>

Your partner, Member B, has made a choice for stage 2. Please select the statement that you believe is more likely:

- **Statement I**: Member B has chosen *In*.

- **Statement O**: Member B has chosen *Out*.

You will earn 5 points if your forecast is correct (i.e. if Member B chooses *In* and you select Statement I, or B chooses *Out* and you select Statement O). You will earn nothing otherwise.

Member B's Forecast Task(s)

Your partner, Member A, has made choices for both stage 1 and 3; also, he or she is selecting between **Statement I** and **Statement O**, each of which is a statement about the choice you just made for stage 2. Which choices do you think your partner A has made for his or her stages, **and** which statement do you think your partner A is selecting?

Notice that A's selections can be expressed in the table below. The column represents A's selection of statement, the row represents A's choices for 1st and 3rd stages. So each cell represents an outcome of A's choices **and** statement. For example, the upper-left cell represents the outcome that A has chosen *Out* for 1st stage, *In* or *Out* for 3rd stage, **and** *Statement I*.

Statement I	■		
Statement O			
	1st Stage *Out*, 3rd *In* or *Out*	1st Stage *In*, 3rd Stage *Out*	1st Stage *In*, 3rd Stage *In*

Your first forecast task

Your first task is to forecast the percent chance that each of the six outcomes happens. A percent chance is a number between 0 and 100, where 100 means that you are certain that such outcome is the correct one, and 0 means that you are certain that such outcome is *not* the correct one. Enter the percent chance of each outcome into the corresponding cell. If you leave any cell as blank it will be viewed as 0. Make sure the six numbers sum up to 100.

You will earn 5 points if your forecast exactly coincide with your partner A's statement **and** choices. If your forecast does not exactly coincide with your partner A's choice and statement, you will receive 5 points minus 2.5 times a penalty amount. The penalty amount is the sum of squared distances between each of the six numbers you entered and the correct answer, i.e. the outcome from A's selection.

Example: Suppose you believe that with 80 percent chance A has chosen to play *In* for 1st and *Out* for 3rd stage, **and** has selected *Statement I*; with 15 percent chance A has chosen to play *In* for 1st and *Out* for 3rd stage, **and** has selected *Statement O*; with

5 percent chance A has chosen to play *In* for 1st and *In* for 3rd stage, **and** has selected *Statement O*, you should enter the numbers as below:

Statement I	0	80	0
Statement O	0	15	5 ■
	1st Stage *Out*, 3rd *In* or *Out*	1st Stage *In*, 3rd Stage *Out*	1st Stage *In*, 3rd Stage *In*

Now suppose your partner A has chosen *In* for 1st and *In* for 3rd stage, and has selected *statement O*. The penalty amount is $(100/100 - 5/100)^2 + (0 - 80/100)^2 + (0 - 15/100)^2 = 1.54$. So you earn $5 - 2.5 * 1.54 = 1.15$ from this forecast.

Your second forecast task

After the computer carries out your partner's and your choices, you will be informed if your partner A has chosen *In* for stage 1. Now you have a chance to make a second forecast. A four-cell table will be presented to you. (The first column of the table in your first forecast task is removed because A has chosen *In* for stage 1.) Please make a percent chance forecast again. Your penalty amount and earning point are calculated in the same way as in your first forecast task.

Final Comments

At the end of this experiment one round will be randomly selected to count for payment. Your earning in each round is the sum of the points you earn from the choice task and the forecast task(s). The exchange rate between points and US dollars is 2.5 : 1. Your cash payment will be your earning in US dollars plus the $5 show-up fee.

Your decisions and your payment will be kept confidential. You have to make decisions entirely on your own. Please do not talk to others. If you have any question at any time, raise your hand and the experimenter will come and assist you individually. Please turn off your cell phone and other electronic devices.

If you have any question, please raise your hand now. Otherwise we will proceed to the quiz.

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[1] R. Aumann and A. Brandenburger. Epistemic conditions for nash equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1161–1180, 1995.

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