Periodic, Complete-participation Trade in the Lagos-Rocheteau Model*

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Abstract

Lagos-Rocheteau (Econometrica, 2009) is part of the literature that applies a search model to asset trade in the over-the-counter market. The only friction in their model is that it takes agents cost to getting into contact with other agents. In this paper, as an alternative to their investor-dealer random meetings, a centralized competitive market, which occurs periodically, is studied. This arrangement preserves the main tension in their paper: a tradeoff between a portfolio that maximizes current utility and one that is good on average. For numerical versions of the model, it is shown that this market must occur only infrequently in order for investors to be as well off as their counterparts in the Lagos-Rocheteau setup.

Keywords: over-the-counter market, random search, periodic market, asset trade.

JEL classification: D40, G1

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1 Introduction

Lagos-Rocheteau (LR) (2009) study a continuous-time economy with non-atomic measures of two kinds of infinitely lived agents, investors and dealers. There are two divisible goods, an asset and a numeraire, and the aggregate quantity of the asset is exogenous. Preferences are quasi-linear. In particular, investors have additively separable preferences over the two goods: they get strictly concave (flow) utility from the asset, which is subject to heterogeneous preference shocks which arrive according to a Poisson process; and they get linear utility from the numeraire. Dealers have the same preferences except that they care only about the numeraire. Investors meet dealers pairwise and at random according to another Poisson process, while each dealer is also in continuous contact with all other dealers. The allocation problem in this economy concerns how the asset is distributed among the investors.

LR is part of a literature that is meant to be a theory of transaction costs (bid-ask spreads) in over-the-counter markets for securities (see Duffie et al. 2005; Vayanos and Weill 2008). However, the model contains none of the frictions that seem to give rise to actual over-the-counter markets. The main frictions would seem to be: the desire to trade large quantities and the attendant concern about the effect on the price of large trades; the fact that trades are agreed to before they are executed, which leads participants to care about the credibility of their counter-parties; and asymmetric information about the securities being traded. Instead, in a meeting between an investor and a dealer in LR, there is a known competitive price of the asset, which the current trade does not affect; trade is *quid pro quo* with immediate execution made possible by the assumed quasi-linear preferences; and there is symmetric information in two respects: the asset is a uniform object and the dealer knows the current preference type and portfolio of the investor. In addition, each such meeting is a one-shot meeting with no possibility of a continuing relationship between the investor and the dealer. The only friction is that an investor is in contact with no one except dealers and that each investor meets a dealer at random. In other words, one meeting, which occurs at random, is free; all others are infinitely costly. This seems an extreme view of securities markets and leads us to consider an alternative specification of costly contacts.

We adopt all the assumptions of LR except the assumption about who is in contact with whom. We assume that there is a centralized competitive market among investors, which, however, occurs only periodically. There are no dealers. For us, the infrequency with which the market occurs is a proxy for the cost of operating a market. Moreover, this market infrequency preserves the main tension in LR: the possible mismatch between an agent's current preference type and the agent's current asset holding. In other words, we use the market infrequency to capture the two frictions in LR: search friction and the bargaining power of the dealer. However, we do not point to provide a more realistic description of over-the-counter markets than the one in the LR model, even though it is a task to be done.

Our model where agents have deterministic periodic access to the competitive market is close to a version of the LR model where the investors have access to a competitive market at some Poisson rate. For example, Lagos and Rocheteau (2006) show that from the point of view of the investors it is as if they are trading in a competitive market infrequently at some Poisson rate. In addition, The bargaining friction in the LR model is formally equivalent to an additional delay in our model. In this case our investor's decision problem is identical to the one in Lagos-Rocheteau (2006). The main difference is that our investors have access to the competitive market at the same time instead of being i.i.d. across investors in Lagos-Rocheteau (2006). This assumption does not affect the investor's decision problem and it does not affect the market clearing condition either. But it does affect the distribution of asset holdings across investors.[1]

For numerical versions of the model, we ask how frequently this market must occur in order for investors to be as well off as they are in the LR trading arrangement. For example, if there is one dealer per 100 investors in the LR model, which implies that an investor meets a dealer once every 10 days on average, then our investors are as well off if our centralized market meets once every 26.4 days. The lower trade frequency does not harm investors because they avoid the dealer pay-off in LR, which arises from their

[1] I need to thank someone here.

assumed Nash bargaining between investors and dealers.

The plan of the paper is as follows. The environment is set out in Section 2. Equilibrium under periodic competitive trade is analyzed in Section 3. In Section 4, we present welfare comparisons between the two trading arrangements. All proofs appear in the Appendix.

2 Environment

Time υ is continuous, starts at $\upsilon = 0$, and goes on forever. There is a unit measure of infinitely lived agents. There is one asset and a numeraire good and preferences are quasi-linear. The asset is durable, perfectly divisible, and in fixed supply $A \in R_+$. Instantaneous utility is $u_l(a) + c$. Here $u_l(a)$ is (flow) utility from the asset, where $a \in R_+$ is quantity of the asset held and $l \in L = \{1,..., L\}$ denotes a preference type. The function $u_l(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave, with $u_l^{'}(0) = \infty$ and $u_l^{'}(\infty) = 0$. Utility from the numeraire is $c \in R$, where c is the net consumption of the numeraire ($c < 0$ if an agent produces more of the numeraire than she consumes). Production and consumption of the numeraire occur instantaneously. The preference type of an agent evolves according to the following stochastic process. There is a preference shock to the consumption of the asset for each agent, which has a Poisson arrival rate δ, and is independent across agents. Conditional on the arrival of the preference shock, the probability that an agent draws preference type l is $\pi_l > 0$, with $\sum_{l=1}^{L} \pi_l = 1$. Given a time path of the asset holding, $a(\upsilon)$, and consumption of the numeraire at times $t \in \{T_{\alpha_1}, T_{\alpha_2}, ...\}$[2], the discounted lifetime utility for an agent with preference type l at time $\upsilon = 0$ is

$$E_l \int_0^{\infty} e^{-r\upsilon} u_{k(\upsilon)}(a(\upsilon))d\upsilon + \sum_{i=1}^{\infty} e^{-rT_{\alpha_i}} c(T_{\alpha_i}), \tag{1}$$

where r is the discount rate, and $k(\upsilon) \in L$ denotes the agent's preference type at time υ. The expectation operator, E_l, is over the random variable $k(\upsilon)$, which is indexed by l to indicate that it is conditional on $k(0) = l$. By assumption, $Pr[k(\upsilon) = j|k(t) = l] =$

[2]Consumption of the numeraire is purely atomic.

$[1-e^{-\delta(\upsilon-t)}]\pi_j + e^{-\delta(\upsilon-t)} I_{\{j=l\}}$, and $\forall \upsilon \geq t$.

To describe the heterogeneity in this economy, let μ be a probability measure on (Z, Z_β), where $Z = L \times R_+$, Z_β is the Borel σ-algebra over Z. Then μ is a probability measure that represents the distribution of agents across asset holdings and preference types. We define a set of joint distributions, $\mu \in \Psi$ that satisfy $\sum_{l=1}^{L} \mu(l, \cdot) = 1$.

3 Competitive equilibrium

Suppose there is a competitive asset market at each $t \in \Lambda \equiv \{0, \Delta, 2\Delta, ...\}$, that is, $T_{\alpha_{i+1}} - T_{\alpha_i} = \Delta$, where Δ captures how frequently the market opens. Denote $p(t)$ as the asset price, and let $a_l(t)$ be the post-trade asset holding for an agent with preference type l at time t.

Before we define an agent's utility maximization problem, it is convenient for us to define $U_l(a_l(t))$. It is the expected discounted utility flow from the asset holding $a_l(t)$ over the time interval $[t, t+\Delta]$ for an agent with preference type l at time t,

$$U_l(a_l(t)) = E_l \int_t^{t+\Delta} e^{-r(\upsilon-t)} u_{k(\upsilon)}(a_l(t)) d\upsilon. \tag{2}$$

We now set out the individual utility maximization problem and label it "problem 1".

Problem 1: Given a time path of price $\{p(t)\}_{t\in\Lambda}$, the utility maximization problem for an agent with preference type l at time $t = 0$, is to choose a time path of the asset holding $\{a_{k(t)}(t)\}_{t\in\Lambda}$ so as to maximize[3]

$$\sup_{\{a_{k(t)}(t)\}_{t\in\Lambda}} \left\{ \begin{array}{c} \sum_{t\in\Lambda} e^{-tr} \sum_{j}^{L} \left\{[1-e^{-\delta t)}]\pi_j + e^{-\delta t} I_{\{j=l\}}\right\} \\ * \left\{U_j(a_j(t)) - p(t)\left[a_j(t) - a_{k(t-\Delta)}(t-\Delta)\right]\right\} \end{array} \right\}. \tag{3}$$

Denote

$$q(t) = \left[p(t) - e^{-\Delta r} p(t+\Delta)\right].$$

[3] Time $t = 0$ is not a market day.

In the Appendix, it is shown that the solution to problem 1, $\{a_l(t)\}_{t\in\Lambda}$, satisfies the following two equations

$$U_l'(a_l(t)) = q(t), \text{ for all } t \in \Lambda, \tag{4}$$

$$\lim_{\upsilon\to\infty} e^{-\Delta r\upsilon} p(\upsilon) a_{k(\upsilon)}(\upsilon) = 0. \tag{5}$$

At each time t, an agent's choice of the asset holding depends on her preference type $k(t)$. The preference type of an agent evolves exogenously. At time t, the measure of agents with preference type l equates to

$$\mu_t(l,\cdot) = \left(1 - e^{-\delta t}\right)\pi_l + e^{-\delta t}\mu_0(l,\cdot). \tag{6}$$

Then the market clearing condition at time t requires that

$$\sum_{l=1}^{L} \mu_t(l,\cdot) a_l(t) = A. \tag{7}$$

We can now define the market equilibrium. It suffices to consider the post-trade distribution μ_t that has finite supports $\{l,\, a_l(t)\}_{l\in L}$.

Definition 1: Given $\mu_0 \in \Psi$, a sequence $\{\mu_t,\, p(t)\}_{t\in\Lambda}$ is an equilibrium if at time t, μ_t has finite supports $\{l,\, a_l(t)\}_{l\in L}$, and $a_l(t)$ is the solution to problem 1 for an agent with preference type l and $\{a_l(t)\}_{l\in L}$ satisfies equation (7).

We will show that there is a unique competitive equilibrium. Since equation (6) implies that $\mu_t(l,\,\cdot)$ converges to π_l exogenously, this economy has a simple steady-state equilibrium.

Proposition 1: There exists a unique competitive equilibrium. For any $\mu_0 \in \Psi$, there exists a unique steady-state equilibrium such that [4]

$$U_l'(a_l^*) = q, \tag{8}$$

$$\sum_{l=1}^{L} \pi_l a_l^* = A. \tag{9}$$

Proposition 1 can be used to derive the steady-state average welfare for this economy. It is a function of Δ, or

$$W(\Delta) = \sum\nolimits_{v=0}^{\infty} e^{-v\Delta r} \sum_{l=1}^{L} \pi_l U_l(a_l^*). \tag{10}$$

Proposition 2: The function $W(\Delta)$ is continuous and weakly decreasing in Δ, or

$$\frac{\partial W(\Delta)}{\partial \Delta} \leq 0. \tag{11}$$

Since agents are subjected to preference shock and not able to adjust asset holdings continuously, there is a misallocation between asset holding and preference type. Equation (11) implies that the misallocation worsens when the market opens less frequently.

4 A welfare comparison with the LR model

In this section, we compare the welfare in our model with that in the LR model. A free entry version of their model is more relevant here as it takes into account the resource cost associated with a dealership system. We first duplicate the equilibrium conditions of the LR model in Section 4.1.

[4]Knowing the sequence $\{\mu_t\}_{t\in\Lambda}$, $\forall v \in (0, \infty)$, for all $j \in L$,

$$\mu_v(j,\, a_l(t)) = \mu_t(l,\, \cdot) \left\{ [1 - e^{-\delta(v-t)}]\pi_j + e^{-\delta(v-t)} I_{\{j=l\}} \right\}.$$

After a steady state is reached,

$$\mu_v(j,\, a_l^*) = \pi_l \left\{ [1 - e^{-\delta(v-t)}]\pi_j + e^{-\delta(v-t)} I_{\{j=l\}} \right\},\ \forall v \in (t, t+\Delta].$$

4.1 Steady-state equilibrium in the LR model

Let $\alpha(\nu)$ be the Poisson arrival rate with which an investor (called an agent in our model) meets a dealer, where ν is the measure of dealers entering the market. Assume that $\alpha(\nu)$ is continuously differentiable. Let $\gamma > 0$ represent the ongoing costs of running the dealership.

Suppose in one[5] of the steady states, a $\widetilde{\nu}$ measure of dealers enters the market, then the measure of investors with asset holding $\widetilde{a}_i$, and preference type j is

$$n_{ij} = \frac{\delta\pi_i\pi_j + \alpha(\widetilde{\nu})\pi_i 1_{\{i=j\}}}{\alpha(\widetilde{\nu}) + \delta}. \tag{12}$$

Average instantaneous utility[6] of the whole economy is

$$\sum_i \sum_j \frac{\delta\pi_i\pi_j + \alpha(\widetilde{\nu})\pi_i 1_{\{i=j\}}}{\alpha(\widetilde{\nu}) + \delta} u_j(\widetilde{a}_i), \tag{13}$$

where $\{\widetilde{a}_i\}$ satisfies the first-order conditions

$$\widehat{u}_i'(\widetilde{a}_i) = q = rp, \tag{14}$$

the market clearing condition

$$\sum_i \pi_i \widetilde{a}_i = A, \tag{15}$$

and the free-entry condition

$$\alpha(\widetilde{\nu})/\widetilde{\nu}\Phi = \gamma, \tag{16}$$

where

$$\Phi = \sum_{i,j} n_{ij}\phi_{ij}, \tag{17}$$

and

$$\phi_{ij} = \eta \frac{\widehat{u}_i(\widetilde{a}_i) - \widehat{u}_i(\widetilde{a}_j) - q\left[\widetilde{a}_i - \widetilde{a}_j\right]}{r + \alpha(\widetilde{\nu})(1-\eta)}, \tag{18}$$

[5] Multiple steady-state equilibria are possible, see LR(2009).
[6] The utility from the numeraire is canceled out.

while η is the bargaining power of a dealer. Here $\widehat{u_i}(\cdot)$ is defined as

$$\widehat{u_i}(a) = \frac{(r + \alpha(\widetilde{\nu})(1-\eta))u_i(a) + \delta \sum_j \pi_j u_j(a)}{r + \alpha(\widetilde{\nu})(1-\eta) + \delta}. \tag{19}$$

Average (steady-state) welfare is

$$\widetilde{W} = \left(\sum_i \sum_j \frac{\delta \pi_i \pi_j + \alpha(\widetilde{\nu}) \pi_i 1_{\{i=j\}}}{\alpha(\widetilde{\nu}) + \delta} u_j(\widetilde{a_i}) - \widetilde{\nu}\gamma \right) \frac{1}{r}. \tag{20}$$

In the case of multiple steady-state equilibria, the steady-state equilibrium with the highest welfare is selected.

4.2 Numerical results

Our welfare comparison is based on the fact that, there is Δ^*, such that,

$$W(\Delta^*) = \widetilde{W}. \tag{21}$$

The existence of Δ^* is based on intermediate value theorem because $W(\Delta)$ is continuous, and non-increasing in Δ. Note that $W(0) > \widetilde{W}$ is obvious. Since no trade is always an option for an agent in the LR model, $W(\infty) \leq \widetilde{W}$.

To do the numerical study, we first parameterize the models. We then ask how frequently our market must occur in order that investors are as well off as they are in the LR setup. We also show how our numerical results vary when we adjust some of the parameters.

Following LR (2009), we normalize the stock of assets by setting $A = 1$. Furthermore, we also let a unit of time correspond to a day and take the rate of time preference to be 10% per year, i.e., $r = 0.1/360$. LR set $\delta = 1/7$ so that investors receive one preference shock every week, on average. And they assume that dealers and investors have equal bargaining power, i.e., $\eta = 0.5$.

There are two unknown functions in the model. We follow LR (2007) and assume that $u_l(a) = \varepsilon_l \frac{a^{1-\sigma}}{1-\sigma}$ and $\alpha(\nu) = v^{\theta}$.

As for the parameters of the utility function, following Kydland and Prescott (1982), and Mehra and Prescott (1985), we use a number between 1 and 2 for σ, that is, $\sigma = 1.5$. We consider the case where $\varepsilon = \{1, 2\}$ and $\pi = \{0.5, 0.5\}$. To determine the parameter of the matching function, we follow Shimer (2005) and let $\theta = \eta$. As, from Hosios (1990), we know that the externality of dealers-entering is internalized if and only if the elasticity of the matching technology $\alpha(\nu)$ coincides with dealers' bargaining power.

What remains is to determine a value for the dealers' ongoing cost. Before we continue, we summarize our choices of parameters in Table 1.

In our numerical study, as Table 2 shows, we first adjust the ongoing cost for dealers such that there is one dealer per 100 investors, as in the steady-state equilibrium of LR (2007). This means that a dealer meets about 10 investors each day. Let Δ_{LR} denote how many days, on average, an investor in the LR model needs to wait before meeting with a dealer. Here $\Delta_{LR} = 10$. We find that $\gamma = 0.442$, and $\Delta^* = 26.406$ satisfy equation (21). Furthermore, the payment to dealers accounts for 55% of the total gain from asset trading. Here the total gain from asset trading is $\widetilde{W} - W(\infty) + \frac{\widetilde{\nu}\gamma}{r}$, and $\frac{\widetilde{\nu}\gamma}{r}$ is the dealer pay-off. We also consider another alternative where there is one dealer per 1000 investors in the steady-state equilibrium of LR (2007). This means an investor meets a dealer once every 31.6 days or $\Delta_{LR} = 31.6$. Then $\gamma = 0.917$, and $\Delta^* = 70.734$. In this case, the cost of dealership accounts for 62% of the total gain from asset trading. We find much lower trade frequency in our model, which is compensated by more than a 50% saving on the dealership cost.

We then conduct some sensitivity analysis. Firstly, as Table 3 shows, given ε_1, changing ε_2 does not significantly change Δ^*, even though ongoing cost γ changes quite a bit. Secondly, as table 4 shows, given $\varepsilon = \{1, 2\}$, changing σ does not significantly change Δ^*. Thirdly, as Table 5 shows, keeping other parameters fixed and maintaining the Hosios condition, changing bargaining power η significantly changes Δ^*. As η rises, we expect γ to increase. However, θ rises as well, which substantially lowers the number of deals a dealer makes per unit of time, as the value of $\alpha(\widetilde{\nu})/\widetilde{\nu}$ shows. The latter effect dominates, so γ decreases.

5 Conclusion

This paper studies periodic, complete-participation trade in the LR model. We argue that our market infrequency preserves the main tension in the LR model: the possible mismatch between an agent's current preference type and the agent's current asset holding. Numerical exercises show that, in order for our agents to be as well off as their counterparts in the LR trading arrangement, there is much lower trade frequency in our model, which is compensated by more than a 50% saving on the dealership cost.

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Appendix

Solving Problem 1

Given price path $\{p(t)\}_{t\in\Lambda}$, let $\{a^*_{k(t)}(t)\}_{t=0}^{\infty}$ satisfy equations (4) and (5), and let $\{a_{k(t)}(t)\}_{t=0}^{\infty}$ be any feasible asset sequence. It is sufficient to show that the difference, denoted as D, between the objective function Problem 1, evaluated at $\{a^*_{k(t)}(t)\}_{t=0}^{\infty}$ and then at $\{a_{k(t)}(t)\}_{t=0}^{\infty}$ is nonnegative. We have

$$
\begin{aligned}
D = {} & U_l(a^*_l(0)) - p(0)a^*_l(0) - [U_l(a_l(0)) - p(0)a_l(0)] + \\
& \lim_{T\to\infty} \sum_{t\geq 1, t\in\Lambda}^{T} e^{-tr} \sum_j^L \left[[1-e^{-\delta t)}]\pi_j + e^{-\delta t} I_{\{j=l\}}\right] \\
& * \left\{ \begin{array}{c} \left[U_j(a^*_j(t)) - p(t)\left[a^*_j(t) - a^*_{k(t-\Delta)}(t-\Delta)\right]\right] \\ - \left[U_j(a_j(t)) - p(t)\left[a_j(t) - a_{k(t-\Delta)}(t-\Delta)\right]\right] \end{array} \right\}.
\end{aligned}
$$

Since utility function $u_i(a)$ is strictly concave, so is $U_l(a)$ and both are continuously differentiable. Hence,

$$
\begin{aligned}
D \geq {} & [U'_l(a^*_l(0)) - p(0)]\,[a^*_l(0) - a_l(0)] + \lim_{T\to\infty} \sum_{t=\Delta}^{T} e^{-tr} \sum_j^L \left[[1-e^{-\delta t)}]\pi_j + e^{-\delta t} I_{\{j=l\}}\right] \\
& * \left[\left[U'_j(a^*_j(t)) - p(t)\right]\left[a^*_j(t) - a_j(t)\right] + p(t)\left[a^*_{k(t-\Delta)}(t-\Delta) - a_{k(t-\Delta)}(t-\Delta)\right]\right].
\end{aligned}
$$

Rearranging terms gives,

$$
\begin{aligned}
D \geq {} & [U'_l(a^*_l(0)) - p(0)]\,[a^*_l(0) - a_l(0)] + \sum_j^L \left[[1-e^{-\delta\Delta)}]\pi_j + e^{-\delta\Delta} I_{\{j=l\}}\right] \\
& * \left\{e^{-\Delta r}\left[U'_j(a^*_j(\Delta)) - p(\Delta)\right]\left[a^*_j(\Delta) - a_j(\Delta)\right] + p(\Delta)e^{-\Delta r}\,[a^*_l(0) - a_l(0)]\right\} \\
& + \lim_{T\to\infty} \sum_{t\geq 2\Delta}^{T} e^{-tr} \sum_j^L \left[[1-e^{-\delta t)}]\pi_j + e^{-\delta t} I_{\{j=l\}}\right] \\
& * \left[\left[U'_j(a^*_j(t)) - p(t)\right]\left[a^*_j(t) - a_j(t)\right] + p(t)\left[a^*_{k(t-\Delta)}(t-\Delta) - a_{k(t-\Delta)}(t-\Delta)\right]\right].
\end{aligned}
$$

With

$$
U'_l(a^*_l(0)) - \left[p(0) - p(\Delta)e^{-\Delta r}\right] = 0,
$$

we have

$$D \geq e^{-\Delta r} \sum_{j}^{L} \left[[1 - e^{-\Delta \delta)}]\pi_j + e^{-\delta \Delta} I_{\{j=l\}} \right] \left\{ \left[U_j'(a_j^*(\Delta)) - p(\Delta) \right] \left[a_j^*(\Delta) - a_j(\Delta) \right] \right.$$
$$+ \lim_{T\to\infty} \sum_{t\geq 2\Delta}^{T} e^{-tr} \sum_{i=1}^{L} \left[[1 - e^{-\delta t)}]\pi_i + e^{-\delta t} I_{\{i=j\}} \right]$$
$$\left. * \left[\left[U_i'(a_i^*(t)) - p(t) \right] \left[a_i^*(t) - a_i(t) \right] + p(t) \left[a_{k(t-\Delta)}^*(t-\Delta) - a_{k(t-\Delta)}(t-\Delta) \right] \right] \right\}.$$

We run the same procedure for each $j \in L$, then

$$\begin{aligned} D \quad \geq \quad & \lim_{T\to\infty} e^{-\Delta T r} \sum_{j}^{L} \left[[1 - e^{-\Delta T \delta)}]\pi_j + e^{-\delta T \Delta} I_{\{j=l\}} \right] * \\ & \left[U_{k(T)}'(a_{k(T)}^*(T)) - p(T) \right] \left[a_{k(T)}^*(T) - a_{k(T)}(T) \right]. \end{aligned}$$

With

$$U_{k(T)}'(a_{k(T)}^*(T)) - p(T) = p(T+\Delta)e^{-\Delta r} \geq 0,$$

this equation leads to

$$\begin{aligned} D \quad & \geq \quad -\lim_{T\to\infty} e^{-\Delta T r} \sum_{j}^{L} \left[[1 - e^{-\Delta T \delta)}]\pi_j + e^{-\delta T \Delta} I_{\{j=l\}} \right] p(T) \left[a_{k(T)}^*(T) - a_{k(T)}(T) \right] \\ & \geq \quad -\lim_{T\to\infty} \sum_{j}^{L} \pi_j e^{-\Delta T r} p(T) a_j^*(T), \end{aligned}$$

where the last line follows from $a_{k(t)}(t) \geq 0$, for all t. It then follows equation (5) that $D \geq 0$, establishing the desired result. Q.E.D.

Proof of Proposition 1

For all t, the distribution of $\{\mu_t(l, \cdot)\}_{l=1}^{L}$ is unique and given by equation (6).

As the utility function $u_i(a)$ is strictly concave, so is $U_i(a)$. Hence, given $\{p(t)\}$, there is a unique solution to equation (4). We denote the solution as $a_l(t)$, which is continuous and strictly decreasing in $q(t) = \left[p(t) - e^{-\Delta r} p(t+\Delta)\right]$, so is $\sum_{l=1}^{L} \mu_t(l, \cdot) a_l(t)$. If we then apply market clearing condition (7), then there is a unique $q(t)$ such that $\sum_{l=1}^{L} \mu_t(l, \cdot) a_l(q(t)) = A$. Given this $q(t)$, there is a unique $\{a_l(t)\}_{l=1}^{L}$ that solves equation (4). This proves the existence of a unique competitive equilibrium.

From equation (6), $lim_{t\longrightarrow\infty} \mu_t(l, \cdot) = \pi_l$ for each l. By a similar argument to the above, there is a unique, time-invariant q that clears the asset market, such that $\sum_{l=1}^{L} \pi_l a_l(q) = A$. Given this q, equation (4) implies a unique set of time-invariant optimal asset holdings $\{a_l^*\}_{l=1}^{L}$. This proves the existence of a unique steady-state equilibrium. Q.E.D.

Proof of Proposition 2

Obviously, $W(\Delta)$ is continuous and differentiable because utility functions are continuous and differentiable. Now let us show that $\frac{\partial W(\Delta)}{\partial \Delta} \leq 0$.

The outline of the proof is as follows. Firstly, we derive another expression of $U_l(a)$, equation (22), and use it to calculate $W(\Delta)$. Secondly, we differentiate $W(\Delta)$ with respect to Δ. We then use equations (8) and (9) to derive another expression of $\frac{\partial W(\Delta)}{\partial \Delta}$, which is equation (24). Thirdly, we show that the first term on the right-hand side of equation (24) is positive, as the integral of a strictly positive function is positive, which is inequation (25). Finally, we show that the second term on the right-hand side of equation (24) is positive, which is inequation (26). We group terms on the right-hand side of inequation (26) by pairs of preference type, e.g., $l,\ j \in L$. For any pair $l,\ j \in L$, we can assume that $a_j^* \geq a_l^*$. It is shown that strict concavity of utility functions imply inequations (28) and (29). Then equation (8) implies that, an agent raises her asset holding only when preference shock raises the agent's current marginal utility, which is inequation (30). Now let us turn to the detail of the proof.

Firstly, changing variables, equation (2) can be written as

$$U_l(a) = E_l \int_0^{\Delta} e^{-rv} u_{k(v)}(a) dv.$$

Combined with

$$Pr[k(v) = j | k(t) = l] = [1 - e^{-\delta(v-t)}]\pi_j + e^{-\delta(v-t)} I_{\{j=l\}}, \forall v \geq t,$$

we have

$$\begin{aligned} U_l(a) &= \int_0^\Delta \sum_j \left\{ \left[1 - e^{-\delta \upsilon}\right] e^{-r\upsilon} \pi_j u_j(a) \right\} d\upsilon + \int_0^\Delta e^{-r\upsilon} e^{-\delta\upsilon} u_l(a) d\upsilon \\ &= \sum_j \pi_j u_j(a) \int_0^\Delta \left[e^{-r\upsilon} - e^{-(\delta+r)\upsilon}\right] d\upsilon + u_l(a) \int_0^\Delta e^{-(\delta+r)\upsilon} d\upsilon. \end{aligned}$$

Denote

$$\overline{u}(a) = \sum_j \pi_j u_j(a),$$

then

$$\begin{aligned} U_l(a) &= \overline{u}(a) \left[-\frac{1}{r} e^{-r\upsilon} + \frac{1}{\delta + r} e^{-(\delta+r)\upsilon} \right]_0^\Delta + u_l(a) \left[-\frac{1}{\delta+r} e^{-(\delta+r)\upsilon} \right]_0^\Delta \\ &= \overline{u}(a) \left[-\frac{1}{r} e^{-r\Delta} + \frac{1}{\delta+r} e^{-(\delta+r)\Delta} - \frac{1}{\delta+r} + \frac{1}{r} \right] + u_l(a) \left[-\frac{1}{\delta+r} e^{-(\delta+r)\Delta} + \frac{1}{\delta+r} \right], \end{aligned}$$

or

$$U_l(a) = \frac{\left[-\frac{\delta+r}{r} e^{-r\Delta} + e^{-(\delta+r)\Delta} + \frac{\delta}{r} \right] \overline{u}(a) + u_l(a) \left[1 - e^{-(\delta+r)\Delta}\right]}{\delta + r}. \tag{22}$$

Secondly, with equation (10),

$$\begin{aligned} \frac{\partial W(\Delta)}{\partial \Delta} = &\; \frac{\sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[-\frac{\delta+r}{r} e^{-r\Delta} + e^{-(\delta+r)\Delta} + \frac{\delta}{r} \right] + u_l'(a_l^*) \left[1 - e^{-(\delta+r)\Delta}\right] \right\} \frac{\partial a_l^*}{\partial \Delta}}{\left(1 - e^{-r\Delta}\right)(\delta + r)} \\ &+ \frac{\sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[(\delta+r) e^{-r\Delta} - (\delta+r) e^{-(\delta+r)\Delta} \right] + u_l(a_l^*) \left[(\delta+r) e^{-(\delta+r)\Delta} \right] \right\}}{(\delta+r)(1 - e^{-r\Delta})} \\ &- r e^{-r\Delta} \frac{\sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[-\frac{\delta+r}{r} e^{-r\Delta} + e^{-(\delta+r)\Delta} + \frac{\delta}{r} \right] + u_l(a_l^*) \left[1 - e^{-(\delta+r)\Delta}\right] \right\}}{\left(1 - e^{-r\Delta}\right)^2 (\delta + r)}. \end{aligned}$$

Since equation (8) can be rewritten as

$$\frac{\overline{u}'(a_l^*) \left[-\frac{\delta+r}{r} e^{-r\Delta} + e^{-(\delta+r)\Delta} + \frac{\delta}{r} \right] + u_l'(a_l^*) \left[1 - e^{-(\delta+r)\Delta}\right]}{\delta + r} = q, \text{ for all } l \in L, \tag{23}$$

we have

$$\frac{\partial W(\Delta)}{\partial \Delta} = \sum_{l=1}^{L} \pi_l q(\Delta) \frac{\partial a_l^*}{\partial \Delta} + e^{-r\Delta} \frac{\sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[1 - e^{-\delta\Delta} \right] + u_l(a_l^*) \left[e^{-\delta\Delta} \right] \right\}}{(1 - e^{-r\Delta})^2} (1 - e^{-r\Delta})$$
$$+ \frac{e^{-r\Delta} \sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[(\delta + r) e^{-r\Delta} - r e^{-(\delta+r)\Delta} - \delta \right] + u_l(a_l^*) \left[r e^{-(\delta+r)\Delta} - r \right] \right\}}{(1 - e^{-r\Delta})^2 (\delta + r)}.$$

With equation (9), we have

$$\sum_{l=1}^{L} \pi_l \frac{\partial a_l^*}{\partial \Delta} = 0,$$

so

$$\sum_{l=1}^{L} \pi_l q(\Delta) \frac{\partial a_l^*}{\partial \Delta} = 0.$$

Therefore

$$\frac{\partial W(\Delta)}{\partial \Delta} = \frac{e^{-r\Delta} \sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[1 - e^{-r\Delta} - e^{-\delta\Delta} + e^{-(\delta+r)\Delta} \right] + u_l(a_l^*) \left[e^{-\delta\Delta} - e^{-(\delta+r)\Delta} \right] \right\}}{(1 - e^{-r\Delta})^2 (\delta + r)}$$
$$+ \frac{e^{-r\Delta} \sum_l \pi_l \left\{ \overline{u}'(a_l^*) \left[(\delta + r) e^{-r\Delta} - r e^{-(\delta+r)\Delta} - \delta \right] + u_l(a_l^*) \left[r e^{-(\delta+r)\Delta} - r \right] \right\}}{(1 - e^{-r\Delta})^2 (\delta + r)}.$$

Then

$$\frac{\partial W(\Delta)}{\partial \Delta} = \frac{e^{-r\Delta} \sum_l \pi_l \overline{u}'(a_l^*) \left[1 - e^{-r\Delta} + e^{-r\Delta} - \frac{\delta}{(\delta+r)} - \left(e^{-\delta\Delta} - e^{-(\delta+r)\Delta} + \frac{r}{(\delta+r)} e^{-(\delta+r)\Delta} \right) \right]}{(1 - e^{-r\Delta})^2}$$
$$+ \frac{e^{-r\Delta} \sum_l \pi_l u_l(a_l^*) \left[e^{-\delta\Delta} - e^{-(\delta+r)\Delta} + \frac{r e^{-(\delta+r)\Delta}}{(\delta+r)} - \frac{r}{(\delta+r)} \right]}{(1 - e^{-r\Delta})^2},$$

or

$$\frac{\partial W(\Delta)}{\partial \Delta} = \frac{e^{-r\Delta} \sum_l \pi_l \overline{u}'(a_l^*) \left[(\delta + r) - \delta - (\delta + r) e^{-\delta\Delta} + (\delta + r) e^{-(\delta+r)\Delta} - r e^{-(\delta+r)\Delta} \right]}{(1 - e^{-r\Delta})^2 (\delta + r)}$$
$$+ \frac{e^{-r\Delta} \sum_l \pi_l u_l(a_l^*) \left[(\delta + r) e^{-\delta\Delta} - (\delta + r) e^{-(\delta+r)\Delta} + r e^{-(\delta+r)\Delta} - r \right]}{(1 - e^{-r\Delta})^2 (\delta + r)}.$$

So

$$\frac{\partial W(\Delta)}{\partial \Delta} = \frac{e^{-r\Delta}\sum_l \pi_l \left\{ \begin{array}{c} \overline{u}'(a_l^*)\left[r-(\delta+r)\,e^{-\delta\Delta}+\delta e^{-(\delta+r)\Delta}\right] \\ +u_l(a_l^*)\left[(\delta+r)\,e^{-\delta\Delta}-\delta e^{-(\delta+r)\Delta}-r\right] \end{array} \right\}}{(1-e^{-r\Delta})^2\,(\delta+r)},$$

or

$$\frac{\partial W(\Delta)}{\partial \Delta} = -\frac{\left[-(\delta+r)\,e^{-\delta\Delta}+\delta e^{-(\delta+r)\Delta}+r\right]e^{-r\Delta}\sum_l \pi_l\left[u_l(a_l^*)-\overline{u}(a_l^*)\right]}{(1-e^{-r\Delta})^2\,(\delta+r)}. \tag{24}$$

We want to show $\frac{\partial W(\Delta)}{\partial \Delta} \leq 0$.

Thirdly, we show that

$$-(\delta+r)e^{-\delta\Delta}+\delta e^{-(\delta+r)\Delta}+r>0, \text{ for all } \delta,\, r,\, \Delta \in R_+.$$

This result comes from

$$\int_0^{\Delta}\sum_j \left\{\left[1-e^{-\delta v}\right]e^{-rv}\pi_j u_j(a)\right\}dv > 0, \text{ for all } \delta,\, r,\, \Delta \in R_+.$$

It implies that

$$-\frac{\delta+r}{r}e^{-r\Delta}+e^{-(\delta+r)\Delta}+\frac{\delta}{r}>0, \text{ for all } \delta,\, r,\, \Delta \in R_+,$$

or

$$-(\delta+r)\,e^{-\delta\Delta}+\delta e^{-(\delta+r)\Delta}+r>0, \text{ for all } \delta,\, r,\, \Delta \in R_+.$$

Changing the positions of r and δ, then

$$-(\delta+r)e^{-\delta\Delta}+\delta e^{-(\delta+r)\Delta}+r>0, \text{ for all } \delta,\, r,\, \Delta \in R_+. \tag{25}$$

Finally, to show that $\frac{\partial W(\Delta)}{\partial \Delta} \leq 0$, it suffices to show that

$$\sum_l \pi_l\left[u_l(a_l^*)-\overline{u}(a_l^*)\right] \geq 0. \tag{26}$$

Select a pair $l,\ j \in L$, without lost of generality, we can assume that $a_j^* \geq a_l^*$. By definition,

$$\begin{aligned}\sum_l \pi_l u_l(a_l^*) &= \sum_j \sum_l \pi_l \pi_j u_l(a_l^*) \\ &= \sum_{l,\ j} \left[\pi_l \pi_j u_l(a_l^*) + \pi_j \pi_l u_j(a_j^*)\right] \\ &= \sum_{l,\ j} \pi_l \pi_j \left[u_l(a_l^*) + u_j(a_j^*)\right],\end{aligned}$$

where $a_j^* \geq a_l^*$ for all pair $l,\ j \in L$. Similarly

$$\begin{aligned}\sum_l \pi_l \bar{u}(a_l^*) &= \sum_l \pi_l \sum_j \pi_j u_j(a_l^*) \\ &= \sum_{l,\ j} \left[\pi_l \pi_j u_j(a_l^*) + \pi_j \pi_l u_l(a_j^*)\right] \\ &= \sum_{l,\ j} \pi_l \pi_j \left[u_j(a_l^*) + u_l(a_j^*)\right],\end{aligned}$$

where $a_j^* \geq a_l^*$ for all pair $l,\ j \in L$. Hence,

$$\begin{aligned}& \sum_l \pi_l \left[u_l(a_l^*) - \overline{u}(a_l^*)\right] \\ = \ & \sum_{l,\ j} \left\{\pi_l \pi_j \left[u_l(a_l^*) + u_j(a_j^*)\right] - \pi_l \pi_j \left[u_j(a_l^*) + u_l(a_j^*)\right]\right\}.\end{aligned} \tag{27}$$

Since

$$u_l^{''}(\cdot) < 0, \text{ for all } l,$$

$$\begin{aligned}& \pi_l \pi_j \left[u_l(a_l^*) + u_j(a_j^*)\right] - \pi_l \pi_j \left[u_j(a_l^*) + u_l(a_j^*)\right] \\ \geq \ & \pi_l \pi_j \left[u_j^{'}(a_j^*)\left(a_j^* - a_l^*\right) + u_l^{'}(a_l^*)\left(a_l^* - a_j^*\right)\right],\end{aligned}$$

or

$$\begin{aligned}& \pi_l \pi_j \left[u_l(a_l^*) + u_j(a_j^*)\right] - \pi_l \pi_j \left[u_j(a_l^*) + u_l(a_j^*)\right] \\ \geq \ & \pi_l \pi_j \left[\left[u_j^{'}(a_j^*) - u_l^{'}(a_l^*)\right]\left(a_j^* - a_l^*\right)\right],\end{aligned} \tag{28}$$

where $a_j^* \geq a_l^*$ for all pair $l,\, j \in L$.

By equation (8) or (23), for all $l,\, j \in L$,

$$
\begin{aligned}
& \frac{\overline{u}'(a_l^*)\left[-\frac{\delta+r}{r}e^{-r\Delta}+e^{-(\delta+r)\Delta}+\frac{\delta}{r}\right]+u_l'(a_l^*)\left[1-e^{-(\delta+r)\Delta}\right]}{\delta+r} \\
= \; & \frac{\overline{u}'(a_j^*)\left[-\frac{\delta+r}{r}e^{-r\Delta}+e^{-(\delta+r)\Delta}+\frac{\delta}{r}\right]+u_j'(a_j^*)\left[1-e^{-(\delta+r)\Delta}\right]}{\delta+r}.
\end{aligned}
$$

Since $a_j^* \geq a_l^*$, then

$$
\overline{u}'(a_j^*)\left[-\frac{\delta+r}{r}e^{-r\Delta}+e^{-(\delta+r)\Delta}+\frac{\delta}{r}\right] \leq \overline{u}'(a_l^*)\left[-\frac{\delta+r}{r}e^{-r\Delta}+e^{-(\delta+r)\Delta}+\frac{\delta}{r}\right]. \tag{29}
$$

The above two equations then imply

$$
u_j'(a_j^*) \geq u_l'(a_l^*), \text{ if } a_j^* \geq a_l^*. \tag{30}
$$

So

$$
\pi_l \pi_j \left[\left[u_j^{'}(a_j^*) - u_l^{'}(a_l^*)\right]\left(a_j^* - a_l^*\right)\right] \geq 0. \tag{31}
$$

For inequation (31), sum over all $l,\, j \in L$, then equation (27) implies

$$
\sum_l \pi_l \left[u_l(a_l^*) - \overline{u}(a_l^*)\right] \geq 0.
$$

Therefore, $\frac{\partial W(\Delta)}{\partial \Delta} \leq 0$. Q.E.D.

Table 1. Parameters of the models.

Parameters	Value	Meaning
A	1	stock of assets
t	a day	unit of time
r	0.1/360	rate of time preference
δ	1/7	rate of preference shock
η	0.5	bargaining power
L	2	number of preference types
π	$\{0.5, 0.5\}$	the distribution of types
ε	$\{1, 2\}$	difference among types
σ	1.5	risk aversion coefficient
θ	0.5	elasticity of matching function

Table 2. Numerical results.

Variables	One dealer per 100 investors	One dealer per 1000 investors
γ	0.442	0.917
Δ_{LR}	10.0	31.6
Δ^*	26.406	70.734
Δ^*/Δ_{LR}	2.64	2.24
$W(\infty)$	-10800	-10800
$\widetilde{W}$	-10787	-10798
$\widetilde{W} - W(\infty) + \frac{\widetilde{\nu}\gamma}{r}$	28.91	5.30
$\frac{\widetilde{\nu}\gamma}{r}$	15.91	3.30
$\frac{\frac{\widetilde{\nu}\gamma}{r}}{\widetilde{W}-W(\infty)+\frac{\widetilde{\nu}\gamma}{r}}100\%$	55%	62%

[7]

Table 3. $(\gamma,\, \Delta^*)$ as a function of ε_2.

ε_2	γ	Δ^*
0.5	0.221	26.406
2	0.442	26.406
3	1.332	26.408
5	3.574	26.406
10	9.942	26.407

[7] γ: ongoing cost for dealers; Δ_{LR}: average waiting days for an investor in the LR model before meeting with a dealer; Δ^*: the trade frequency in our model; $W(\infty)$: average (steady-state) welfare if there is no trade in our model; $\widetilde{W}$: average (steady-state) welfare in the LR model; $\frac{\widetilde{\nu}\gamma}{r}$: the dealer pay-off in the LR model; $\widetilde{W} - W(\infty) + \frac{\widetilde{\nu}\gamma}{r}$: the total gain from asset trading in the LR model.

[8]

Table 4. $(\gamma,\, \Delta^*)$ as a function of σ.

σ	γ	Δ^*
1.10	0.602	26.396
1.25	0.530	26.398
1.50	0.442	26.406
1.75	0.379	26.409
2.00	0.332	26.427

[9]

Table 5. $(\gamma,\, \Delta^*)$ as a function of η.

η	γ	Δ^*	Δ_{LR}	$\frac{\alpha(\widetilde{\nu})}{\widetilde{\nu}}$
0.25	0.458	7.2	3.2	31.6
0.50	0.442	26.4	10.0	10.0
0.55	0.352	34.8	12.6	7.9
0.60	0.263	46.7	15.8	6.3

[10]

[8] ε_2: utility shock for preference type 2; γ: ongoing cost for dealers; Δ^*: the trade frequency in our model.

[9] σ: risk aversion coefficient; γ: ongoing cost for dealers; Δ^*: the trade frequency in our model.

[10] η: the bargaining power of a dealer; γ: ongoing cost for dealers; Δ^*: the trade frequency in our model; Δ_{LR}: average waiting days for an investor in the LR model before meeting with a dealer; $\frac{\widetilde{\nu}\gamma}{r}$: the dealer pay-off in the LR model.