Why The House Sector Leads The Whole Economy: the Importance of Collateral Constraints and News Shocks*

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Abstract

This paper establishes a dynamic stochastic partial equilibrium model for explaining residential investment dynamics in the United States, focusing on the distinctive cyclical features of residential investment in that it leads the whole economy. This paper is different from the existing literature in that it adds three new features to the model: news shocks, collateral constraints and agent heterogeneity. The partial equilibrium analysis where interest rates are exogenously fixed shows that these assumptions are essential to generating the dynamic pattern in which residential investment leads consumption and the Gross Domestic Product (GDP).

JEL classification: E25,E21,E32

Keywords: news shocks, heterogeneous agents, house sector, collateral constraints, aggregate uncertainty

1 Introduction

For a long time, the housing sector has led the entire economy in business cycles. This happens not only in the United States but also in other countries, including Japan and the U.K. Why does the housing sector lead the whole economy? What does its leadership reveal about the structure of the economy and consumer behavior? This paper proposes a theoretical model to reflect the way that residential investment leads consumption and the Gross Domestic Product (GDP).

Housing is an important factor in business cycles. It contributes around 50 percent of the aggregate wealth per household, according to Greenwood and Hercowitz (1991) and Bertaut and Starr (2000). The variations in house values generate large wealth effects on consumer choices regarding consumption and investment. For example, Case, Quigley, and Shiller (2005) indicate that house prices have a statistically significant and rather large positive effect on household consumption. Using micro data, Campbell and Cocco (2007) confirm the significant response of household consumption to house prices. Similarly, Black, de Meza, and Jeffreys (1996) indicate that a 10 percent increase in house prices leads to a 5 percent increase in the number of small firms in England because of the relaxed financial constraints introduced by the rise of collateral values (house prices). Moreover, residential investment is a good predictor of economic recession. In the past fifty years, eight of ten recessions (including the most recent one) were preceded by a severe reduction in residential investment, as reported by Leamer (2007). Therefore, if we hope to explain and quantify business cycles, we cannot ignore the role played by the housing sector. Leamer (2007) stresses that the housing sector is very important to economic recessions, and any attempt to understand business cycle needs to focus on housing investments in particular.[1]

[1] In Leamer (2007), household investment includes residential investment and consumer durables. The

One must take two steps to fully understand the effects of the housing sector on the economy. The first step is to study how consumers choose between consumption and house purchases during business cycles. The second is to explore how the changes in house prices generated by consumer choices will affect current investment and, hence, future output through the credit market. Our paper contributes to this research area by focusing on the first step only. We study the quantity changes, rather than price changes, in the housing sector. This strategy not only simplifies our analytical framework, but it also helps clearly display the mechanism through which consumers adjust their demand for houses.

The quantity changes in the housing sector (i.e., residential investment) lead GDP, whereas consumption coincides with GDP. Figure 1 plots detrended data for consumption, residential investment and GDP. Clearly, residential investment is about one or two quarters ahead of consumption and GDP. Consumption coincides with GDP. Green (1997) uses the Granger-Causality test to determine that residential investment leads consumption and GDP significantly, whereas neither consumption nor GDP leads residential investment. The purpose of our paper is to quantitatively explain this trend. Our mechanisms and quantitative analysis help us to understand how consumers make choices between consumption and house purchases in business cycles. We consider it essential to explore the roles played by the housing sector in business cycles before moving forward.

In this paper, three key assumptions help to explain the dynamics of residential investment. First, collateralized consumer loans, such as mortgages, are less restricted in size and carry lower interest rates than do unsecured consumer loans, such as credit card debt. This assumption is consistent with the data for the U.S. financial market. In 2002, the 30-year mortgage interest rate in the U.S. was 6.40%, while the average interest rate

data from National Income and Product Accounts(NIPA) shows that residential investment not only leads gross domestic product more but also has higher volatility than consumer durables.

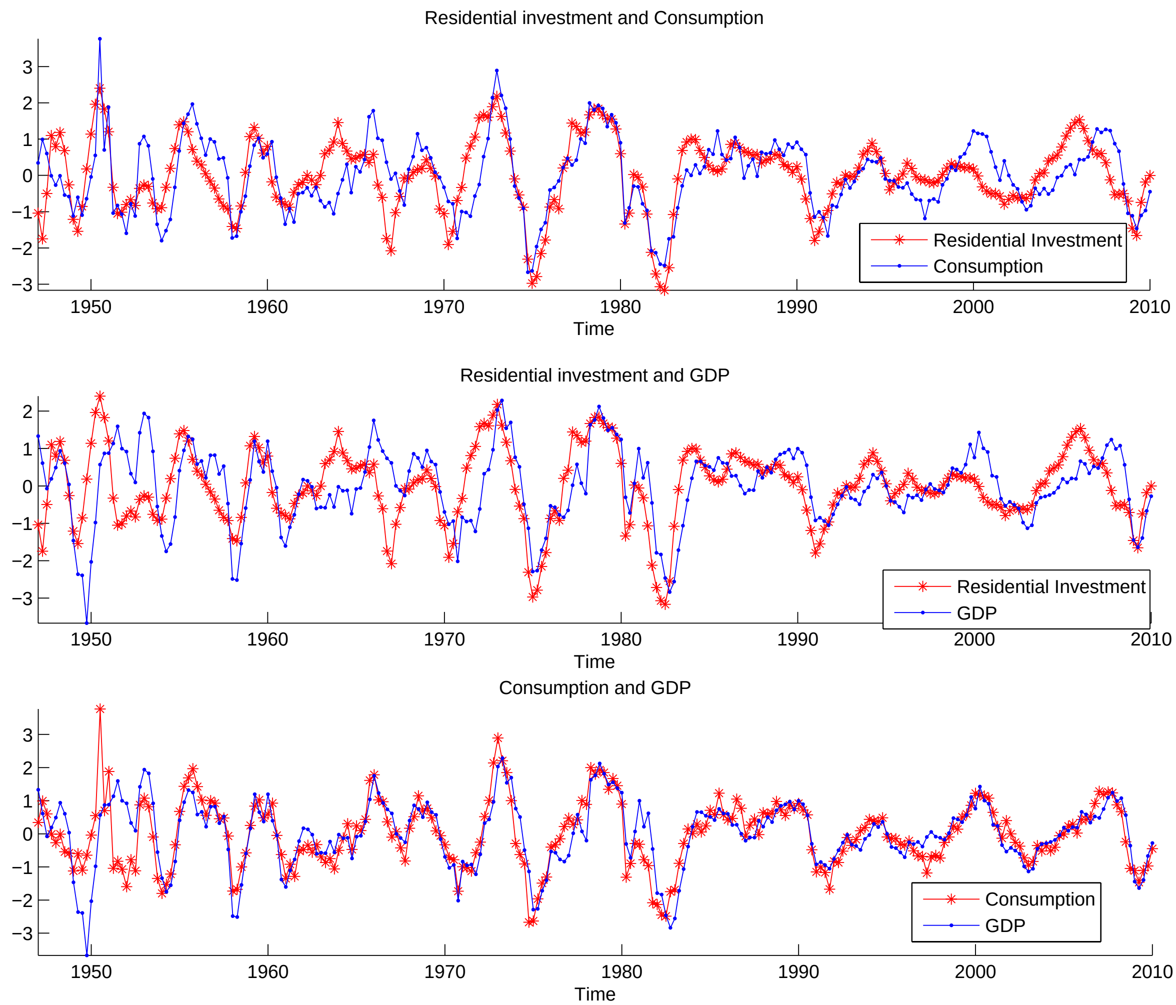

Figure 1: Consumption, Residential Investment, and GDP (1959:Q1-2008:Q4)

for credit card debt was 16.6%.[2] This suggests that, even if unsecured consumer loans are available to everyone, the high cost of borrowing will keep most consumers from using them as a major financing source. In 2001, 81.5% of consumer loans were collateralized by residential properties, whereas unsecured consumer loans amounted to only 10%. [3]

Secondly, we assume that the agents receive noisy news about future total factor productivity (TFP). The news shocks change expectations about the future and hence affect the consumption and investment choices of consumers. This idea was proposed in the early literature by researchers such as Pigou (1927), and has begun to recently attract more attention. People find that contemporaneous shocks to technologies, money, oil prices and credit can account for only a small part of the variation in output [Cochrane (1994); Beaudry and Lucke (2009)]. Beaudry and Portier (2006) provide the empirical evidence that equity prices predict TFP growth in the long term. The shocks to stock prices explain a substantial fraction of the output fluctuations. This gives support to the idea of news shocks as an important resource for business cycles. However, in the neoclassical growth models, news shocks cannot generate booms or even co-movement between employment, consumption, investment and output [for instance, Cochrane (1994); Beaudry and Portier (2004); Beaudry and Portier (2007)]. Jaimovich and Rebelo (2009) propose a theoretical model within this framework and solves these problems by introducing three elements: variable capital utilization, adjustment costs to investment and a weak short-run effect of wealth on the labor supply. Schmitt-Grohe and Uribe (2008) and Khan and Tsoukalas (2009) perform Bayesian estimations of this theoretical model and further confirm that anticipated shocks explain more than half of the predicted aggregate fluctuations. Our paper contributes to this body of literature by applying this assumption to the model of

[2] The data on the mortgage rate is taken from International Monetary Fund (2002). The data on the credit card rate is taken from Gross and Souleles (2002).

[3] The education loans constitute about 50 percent of the unsecured loans. These statistics are taken from Aizcorbe, Kennickell, and Moore (2003) computing these numbers with the data of Survey of Consumer Finance (SCF, 2001).

heterogeneous agents and housing sectors. In this way, we can explore how consumers at different wealth levels consume and make house purchases when the same news shocks affect their expectations about the future.

Thirdly, we use the perpetual model proposed by Blanchard (1985) to explore the effects of income and news shocks on consumer choices regarding house purchases. In this model, young agents are born with an inheritance left to them by the old who have just passed away. They receive endowments during each period and determine their consumption and house purchases. They derive less utility from houses when they become old than when they are young. Hence, consumers are heterogeneous in terms of income, accumulated wealth and preferences regarding houses. This setup helps us to examine and quantify how wealth accumulation and life cycles are related to house purchases.

Through numerical experiments, we find that collateral constraints and news shocks are both essential to generating the leadership of the housing sector over business cycles. Once they have been hit by the news shocks, the agents change their expectations about their future income and hence think differently about current house purchases and consumption. Financial constraints then restrain agent choices because the news shocks about income changes take some time to become reality, and, in the meantime, current incomes remain the same. During this process, wealth heterogeneity plays an important role in the mechanism through which the collateral constraints amplify the response of house purchases to news shocks. Consumers at heterogeneous wealth levels adopt different policies in response to the news shocks: the degree to which they adjust both consumption and housing purchases decreases with wealth levels. In particular, agents with a low wealth level adjust their housing purchases more than their consumption because of collateral constraints. Furthermore, agents accumulate wealth but demand fewer houses as they age. This guarantees that most house purchases are made by collaterally constrained agents who are mostly young and who have a small amount of wealth at their disposal. Hence, the collateral constraints and the news shocks interact with the agent heterogeneity in generating the housing sector's leadership of the economy. Through this mechanism, news shocks can generate much deeper effects on aggregate volatility when combined with financial constraints than without them. This point is not given sufficient attention in the current literature on news shocks. Moreover, wealth heterogeneity also affects the business cycles containing news shocks even when all agents have the same information about the future, and this consideration has not been studied yet.

Our paper also contributes to the large body of literature about economies composed of heterogeneous agents that involve aggregate uncertainty. Some studies focus on the interaction between the collateral constraints, the housing sectors and aggregate dynamics. For example, Iacoviello (2005) sets up a monetary business cycle model with heterogeneous agents, savers (patient workers) and borrowers (impatient entrepreneurs), with collateral constraints tied to house values. Hercowitz and Campbell (2005) use a similar borrower-saver model and finds that the reduced collateral constraints can explain the decline in aggregate volatility since the 1980s. In terms of the details of our model, our study is more similar to the work of Silos (2007), Kiyotaki, Michaelides, and Nikolov (2007) and Fisher and Gervais (2007). These authors develop and quantify a life cycle model of consumption and house purchases under the assumption of aggregate income shocks. Our paper contributes to this literature by providing an analysis of the effects of the news shocks on the agents' behavior, expanding the available financial tools from mortgages alone to both mortgages and credit cards, exploring how wealth affects purchase polices under expectation shocks, and quantitatively showing that residential investment leads the economy.

Our paper is also related to the body of literature that studies neoclassical growth models for housing sectors. These studies seek to match the co-movements between consumption, business investment, residential investment and output. They include papers by Greenwood and Hercowitz (1991), Benhabib, Rogerson, and Wright (1991), Gomme, Kydland, and Rupert (2001), Davis and Heathcote (2005), Fisher (2007) and Gomme and Rupert (2007). The common features of these papers are that they propose dynamic stochastic general equilibrium models of homogeneous agents and that they consider contemporaneous shocks. Gomme and Rupert (2007) are the most recent contributors to this body of literature and proposes carefully calibrated two-sector models. They find that the stochastic growth models of contemporaneous shocks fail to indicate the leadership of residential investment over GDP and consumption.

The structure of the paper is as follows. In section 2, we provide the detailed setup of the model and define the equilibrium. In section 3, we calibrate and parameterize the model. Section 4 displays the numerical results from the experiments and compare different models in order to highlight the importance of the key assumptions made in our model. Section 5 concludes this paper, and discusses some directions of future research.

2 Model

2.1 Agents

The economy consists of a continuum of agents maximizing their expected lifetime utility. Constant population of new generation of the agents are born in each period while the same size of population die. Each new-born agent receives identical heritage from the passed away. The agents live for N ages and have different income levels at different ages of their lives. We use i to denote the age of life, where $i \in \mathcal{N} = \{1, 2, 3, , ...N\}$. For any agent $j \in [0,1]$ who stays at the age of i, the objective function is defined by

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t^j, h_t^j, i_t)\right], \tag{1}$$

where $\beta \in [0,1)$ is the discount factor; $c_t^j \geq 0$ is the consumption for agent j at time t; $i_0 = i$ and $h_t^j \geq 0$ is the self-owned house. The period utility satisfies the following CES form

$$U(c,h,i) = (1-\kappa_h^i)\frac{c^{(1-\theta)}-1}{1-\theta} + \kappa_h^i\frac{h^{(1-\theta_h)}-1}{1-\theta_h},$$

where $\kappa_h^i \in [0,1)$ describes the agent's preferences for the houses and κ_h^i decreases with i. We make this assumption to catch the realistic differences between the young and the old in their demand for the houses. The young people need houses to build the families. And the old people derive less utility from the houses than the young because the family size shrinks with their ages. [4]

At each period, the agent receives an endowment consisting of two parts: individual income and aggregate income. Therefore, his/her budget constraint is

$$c_t^j + h_t^j + a_{t+1}^j \leq a_t^j + h_{t-1}^j(1-\delta_h) + \chi(h_t^j, h_{t-1}^j) + \epsilon_{i_t,t}^j z_t + \mathcal{I}(a_t^j, h_{t-1}^j),$$

where a_t^j is the financial asset; h_{t-1}^j is the house value held in the previous period; δ_h is the depreciation rate of housing capital; χ denotes the adjustment costs for the houses; $\epsilon_{i_t,t}^j$ is

[4] It is the service flow of the housing capital that contributes to the personal utility. However, given the two assumptions of Cob-Douglas home production and CES utility function, we can transform the utility function into the above form. For example, suppose that the utility function satisfies

$$\mathcal{U}(c,c_h,i) = (1-\kappa_h^i)\frac{c^{(1-\theta)}-1}{1-\theta} + \kappa_h^i\frac{c_h^{(1-\theta)}-1}{1-\theta},$$

where c_h is the service flow from the housing capital. Assume that service flow c_h produced by the house h satisfies Cob-Douglas form: $c_h = h^{\alpha_h}$ where α_h is the parameter of the production function. Then by putting this home production function back into the above equation, we can obtain the CES utility function alike and the risk aversion for housing satisfies $\theta_h = 1-(1-\theta)\alpha_h$.

the t-period individual income of the agent j at age i_t; z_t is the aggregate income shock; and $\mathcal{I}$ denotes the return from the liquid wealth (financial asset and housing capital). If $a_t^j < 0$, the agent j borrows a_t^j amount of consumer debts; if $a_t^j > 0$, the agent j saves a_t^j amount of financial assets. The functional form of the adjustment costs for houses is set into the following

$$\chi(h_t^j, h_{t-1}^j) = \begin{cases} 0 & \text{if } h_t^j = h_{t-1}^j \\ -\phi h_{t-1}^j & \text{if } h_t^j \neq h_{t-1}^j \end{cases},$$

which means that if the house value changes, the agent has to pay some constant percentage of the original house value as the adjustment costs. This assumption captures the feature that people spend more time or money to sell expensive houses than the cheap ones.

The individual income of the agent j, $\epsilon_{i,t}^j$, depends on his/her age i. We derive our analytical framework from the perpetual-youth model proposed by Blanchard (1985). At each period, the agent stays at the current age or moves to the next age profile with some possibility until death. We denote the transition matrix of age as $\mathcal{P}^n$. We assume that the agents of the economy do not care about the welfare of their heirs, i.e. no intergenerational altruism. Besides the age profile of the income, the agent also has the idiosyncratic income shocks which are subject to AR(1) process. We denote the log value of the individual income as $e_{i_t,t}^j = \log(\epsilon_{i_t,t}^j)$. $e_{i_t,t}^j$ satisfies the following

$$\begin{aligned} e_{i_t,t}^j &= \zeta i_t + \xi_t^j \\ \xi_t^j &= \varrho \xi_{t-1}^j + \upsilon_t^j \\ \upsilon_t^j &\sim \text{ i.i.d. } \mathrm{N}(0, \sigma_\upsilon^2), \end{aligned}$$

where ζ denotes the growth rate of income over age (or you can also understand it as experiences); i_t denotes the age of the agent j at t period; ξ_t^j denotes the idiosyncratic income; and υ_t^j denotes the innovation to the process of the idiosyncratic income with variance σ_υ^2. Under this assumption of income process, all the agents have the same income growth rates over ages. Besides that, they also have persistent idiosyncratic income shocks. This income process is widely applied in the research of macroeconomics about life-cycle consumption.

In order to make the computation feasible, we need to discretize the process of idiosyncratic income shocks and approximate the conditional expectation by the finite set of its values at each element of the state space. We use the method proposed by Rouwenhorst (1995) to discretize the process of idiosyncratic income shocks ξ. This method has the advantage of keeping the variance and the autocorrelation of the discretized process consistent with the original process. After discretization, we denote the set of finite states as Ξ and the transition probability as $\mathcal{P}^\xi$.

The aggregate productivity shock z_t and the individual income $\epsilon_{i,t}^j$ are two Markov processes, independent with each other. Section 2.3 provides the details about the process of the aggregate productivity z.

2.2 Market Arrangement

There are two forms of consumer debts available in the financial market. The first form of consumer debt is mortgage, of which the interest rate is denoted by r. But the mortgages need to be collateralized by the house values. We use γ to denote the percentage of the house values agents can use as the collateral. It implies that the agents can only borrow up to $\gamma \in [0, 1)$ percent of their house values through the mortgages. The second form is credit card debt, for which the agents need to pay a very high interest rate $r^c \gg r$. We denote h as the value of the houses. Then, the debts smaller than γh are charged with the interest rate r and the debts exceeding γh are charged with credit card rates. The interest rates r and r^c are exogenously given and fixed over time.

Given the above arrangements of financial markets, the capital return $\mathcal{I}$ in the budget constraint is given by

$$\mathcal{I}(a_t^j, h_{t-1}^j) = \begin{cases} ra_t^j & \text{if } a_t^j \geq -\gamma h_{t-1}^j \\ -r\gamma h_{t-1}^j + (a_t^j + \gamma h_{t-1}^j)r^c & \text{if } a_t^j < -\gamma h_{t-1}^j. \end{cases}$$

If the consumer does not borrow more than γh_{t-1}^j, he/she pays or receives the interest rates r. If the consumer borrows more than the value of collateral γh_{t-1}^j, he/she has to pay for two levels of interest rate costs. For the part of secured consumer debts (or financial assets) $-\gamma h_{t-1}$, the agent pays (or receives) a low interest rate r. For the part of unsecured loans $a_t^j + \gamma h_{t-1}^j$, the agent pays a high interest rate r^c. We assume that the interest rates r and r^c are fixed over time and make the quantitative analysis under the framework of partial equilibrium. This set up reduces the costs of computation a lot without hurting the fulfillment of the purpose of our paper.

2.3 News Structure

At period t, the agents receive the signal s_t about z_{t+1}. z_{t+1} contains the information of aggregate income shocks at period $t+1$.

We denote z_t as the current productivity, z_{t+1} as the future productivity and s_t as the current signal. z_t, z_{t+1} and s_t satisfy the following conditions:

$$z_{t+1} = \rho z_t + \nu_{t+1}$$
$$s_t = \rho z_t + \eta_t.$$

The disturbances (ν_{t+1}, η_t) are i.i.d processes and satisfy the following conditions:

$$(\nu_{t+1}, \eta_t) \perp z_t$$
$$(\nu_{t+1}, \eta_t) \sim N(0, \Pi),$$

where Π is defined by

$$\Pi = \sigma_\nu^2 \begin{bmatrix} 1 & \pi \\ \pi & 1 \end{bmatrix}.$$

σ_ν denotes the standard deviation of the disturbances. π denotes the correlation coefficient of η_t and ν_{t+1}. π describes the accuracy of the signal. If $\pi = 1$, the disturbance η_t of the signal is highly correlated with the disturbance ν_{t+1} of the future productivity. The signal contains full information about the future productivity. If $\pi = 0$, η_t is uncorrelated with ν_{t+1}. The signal contains no more information than the current productivity does. Therefore, the higher π is, the more information the signal contains about the future productivity.

We make this assumption about the news structure for the following two reasons. Firstly, although a large literature displays and stresses the forecasting powers of several kinds of signals, it is still far from reaching the agreement on which one should be the best one. Therefore, we do not have any empirical literature estimating the parameter about the accuracy of the news. Secondly, the purpose of our paper is to show the mechanism through which the news affects the choices of the consumers. Hence, this setup is easy for us to display and compare the numerical results of different levels of π.

In order to make the computation feasible, we need to discretize the states of the above two stochastic processes. We do so by extending the method proposed by Tauchen (1986) from one dimension to two dimensions. Appendix A.2 describes in details the method of discretization used in this paper. After discretization, we denote $\mathcal{Z}$ as the set of aggregate income shocks. The set of signal states is denoted by $\mathcal{S}$ and satisfies $\mathcal{S} = \mathcal{Z}$. We denote the transition matrix between (z, s) to (z', s') as Θ.

2.4 Definition of Recursive Partial Equilibrium

Given the interest rates (r, r^c), the recursive partial equilibrium is composed of state space, value function, policy functions, and distribution function. The state space is as encompassing housing capital $h \in \mathcal{H}$, the wealth $w \in \mathcal{W}$, the present idiosyncratic income shock ϵ, the aggregate income shock z, and the signal s for z'.

We denote i as the age of an agent and ξ as the state of his/her idiosyncratic income. His/Her value function $\mathcal{V}$ satisfies the following conditions. If $i < N$, then

$$\mathcal{V}(h, w, i, \xi, z, s) = \max_{\{c, h', a'\}} \left\{ U(c, h', i) + \beta E_{\{i', z'\}}[\mathcal{V}(h', w', i', \xi', z', s') | i, z, s] \right\},$$

subject to:

$$c + h' + a' \leq w(1 + r) + \chi(h', h) + \epsilon_i^{\xi} z \tag{2}$$

$$\chi(h', h) = \begin{cases} -\phi h & \text{if } h' \neq h \\ 0 & \text{if } h' = h \end{cases} \tag{3}$$

$$w' = \begin{cases} a' + h'\frac{1-\delta_h}{1+r} & \text{if } a' \geq -\gamma h' \\ (a' + \gamma h')\frac{1+r^c}{1+r} - \gamma h' + h'\frac{1-\delta_h}{1+r} & \text{if } a' < -\gamma h' \end{cases} \tag{4}$$

$$w'r \geq -B_i. \tag{5}$$

If $i = N$, the agent retires and his/her value function is defined by

$$\mathcal{V}_o(h, w, z, s) = \max_{\{c,h',a'\}} \left\{ U_o(c, h) + \beta E_{\{z'\}}[\mathcal{V}_o(h', w', z', s')|z, s] \right\},$$

subject to:

$$c + h' + a' \leq w(1 + r) + \chi(h', h) + \epsilon_o z$$

$$w'r \geq -B_o,$$

where w' is updated as described by equation (4).

Equation (2) defines the budget constraint. Equation (3) defines the costs of house adjustment. Equation (4) defines the wealth of the next period w'. Equation (5) guarantees that the agents can pay the interest rates of their debts even if they have the lowest income in the next period. This means that the agents cannot borrow more than the amount that their lowest possible wage income can support, based on the cost of the interest rates associated with the consumer loans. A large literature has applied the same assumption in the models of the heterogeneous agent, for example, Aiyagari (1994). The choice variables are the consumption c, the current house h', and the future financial asset a'. Therefore, the policy functions can be denoted as the following: the current consumption $c = g_c(h, w, i, \xi, z, s)$; the current house $h' = g_h(h, w, i, \xi, z, s)$; and the future wealth $w' = g_w(h, w, i, \xi, z, s)$.

λ denotes the distribution of all agents over the state variables (h, w, i, ξ), which is updated each period. Given the states of the next period (h', w', i', ξ'), if the set $\mathcal{B}$ is defined as

$$\mathcal{B} = \{(h, w, i, \xi) \in \mathcal{H} \times \mathcal{W} \times \mathcal{N} \times \Xi | h' = g_h(h, w, i, \xi, z, s), w' = g_w(h, w, i, \xi, z, s)\},$$

the updated distribution function satisfies

$$\lambda'(h', w', i', \xi') = \int_{\mathcal{B}} \lambda(h, w, i, \xi) \mathcal{P}^{\xi}(\xi'|\xi) \mathcal{P}^{n}(i'|i) dh dw di d\xi.$$

3 Calibration and Parametrization

In this model, the set of parameters includes preferences $\{\mathcal{N}, (\kappa_h^i)_{i=1}^N, \theta, \theta_h, \beta\}$, market $\{r, r_h, \delta_h, \gamma, \phi, (B_i)_{i=1}^N\}$, and productivity $\{\zeta, \mathcal{P}^n, \varrho, \sigma_\upsilon^2, \pi, \rho, \sigma_\nu\}$. Table 1 displays the parameters used in the simulations. We will explain how to determine the values of these parameters in this section.

Because first-time home buyers are around the age of 30, we set the life cycle at 35. In addition, we set the number of life stages as $N = 4$, with $i = 1$ corresponding to $36 \sim 45$, $i = 2$ to $46 \sim 55$, $i = 3$ to $56 \sim 65$ and $i = 4$ to 65 and plus. We assume that $\kappa_h^i = \kappa_h^j = \kappa_h$ (if $i, j < 4$), i.e. the agents younger than 65 years old have the same preferences. For the agents older than 65, we set $\kappa_h^4 = 0$ which means that older people do not receive any utility from their houses. Given that the houses can depreciate over time in the model, this assumption guarantees that the older people hold financial assets but not houses. We calibrate κ_h to match the mean of the ratios of residential investment to personal domestic consumption in the U.S. data (1965-2007).

The after-tax yearly mortgage rate is 4.8%. Under the U.S. tax system, mortgage payments can be taken as income tax deductions. With the capital tax rate set at 29.2%, this means that the mortgage rate before tax is 6.7%. We use the value of the capital tax rate from Gomme and Rupert (2007). The after-tax credit card rate is set at 12.1%, which implies that the credit card rate before tax is 17.1%. The depreciation rates of the houses δ_h are chosen as the mean of the ratios of residential investment to the aggregate value of houses (1965-2007). γ is calibrated such that the ratio of aggregate house value over GDP is consistent with the mean of the data. Because ϕ measures the adjustment costs of houses, its value affects how often the agents move. We calibrate ϕ to match the volatility of the residential investment.

The parameter ζ measures the growth rate of income over age. We set ζ at 0.6, which implies that agents' income increases by 60% within ten years. For the AR(1) process for the idiosyncratic income shocks ξ, we choose the autocorrelation coefficient ϱ as 0.98 and σ_v as 0.015. The group $i = 4$ corresponds to those individuals more than 65 years old. We assume that they retire and receive 50% of their average former income as their pension. Appendix A.3 lists the parameters of the stochastic process for individual income. Because the older people have a lower effective discount factor (because of the probability of death), they may choose to reach the lower bound of borrowing because of the relative low risk associated with aggregate volatility. Hence, we set $B_4 = 0$ to avoid implying that the older people might leave a negative amount of wealth to their descendants after death. The aggregate income process z follows the process for the HP filtered real GDP from NIPA (1959:I to 2008:IV). Please refer to Appendix A.3 for the parameters of the aggregate income shocks.

Table 1: Parameters

Preferences	
$\beta = 0.985$	discount factor
$\kappa_h = 0.261$	the weight of housing in utility function
$\theta = 1$	risk aversion for consumption
$\theta_h = 1$	risk aversion for self-owned housing
Market	
$r = 4.80\%$	mortgage rate
$r^c = 12.30\%$	credit card rate
$\delta_h = 0.015$	depreciation rate of the houses
$\gamma = 0.820$	$1 - \gamma$: down payment
$\phi = 0.081$	adjustment costs
$\{B_i\}_{i=1}^N = \{0.035, 0.056, 0.09, 0.00\}$	borrowing constraints
Productivity	
$\zeta = 0.6$	income growth rate over ages
$\varrho = 0.98$	autocorrelation of idiosyncratic income process
$\sigma_\upsilon^2 = 0.015$	the variance of innovation to idiosyncratic shocks
$\rho = 0.82$	autocorrelation of aggregate income process
$\sigma_\nu = 0.017$	standard deviation of innovation to aggregate income process

4 Numerical Results

We compute and compare three economies. In Benchmark economy, the agents have access to mortgages and credit cards. They also receive news about next-period productivity with the correlation of the two disturbances π equal to 0.8. The agents in Economy I face the same financial market structures. However, we set $\pi = 0.2$, which means that the news is much less informative in this economy than in Economy I. In Economy II, $\gamma = 0$ and $\pi = 0.8$, which implies that the agents face the same borrowing constraints for both housing purchases and consumption. Table 2 displays the second moments which are computed with the data simulated using the three models. Table 3 displays the results of Granger Causality test with the real data and the simulated data. In Benchmark Economy, the correlation coefficients of residential investment with GDP reach the highest point at x_{t-1}, implying that residential investment is most closely correlated with future GDP. The Granger-Causality test shows that residential investment Granger-causes consumption and output, whereas neither consumption nor output Granger-causes residential investment. The correlation coefficients of consumption and GDP reach the highest point at x_t, whereas a Granger-Causality test shows that consumption has no Granger-causality relationship with GDP. The Granger-Causality test confirms that the Benchmark Economy displays the same pattern of lead-lag relationship as the real data. In both Economy I and Economy II, residential investment has the highest correlation coefficient with the current GDP. The Granger-causality test also confirms that residential investment does not granger-cause consumption or GDP. These simulation results display that the two assumptions of news shocks and mortgage are both necessary to generate the leadership of residential investment over the whole economy.

A simple example can be used to illustrate the main mechanism at work in this paper. An agent receives good news about future productivity shocks and wants to increase

Table 2: Second Moments of Data and Simulated Results

Variable x	Standard deviation	Correlation coefficients of GDP_t with x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
		Data								
GDP	0.02	0.19	0.38	0.62	0.82	1.00	0.82	0.62	0.38	0.17
Consumption	0.01	0.19	0.38	0.60	0.78	0.88	0.80	0.65	0.46	0.26
Residential Investment	0.10	0.38	0.54	0.68	0.80	0.77	0.63	0.44	0.24	0.05
		Benchmark Economy: $\gamma = 0.82$ **and** $\pi = 0.8$								
GDP	0.02	-0.03	0.39	0.61	0.81	1.00	0.81	0.61	0.39	0.17
Consumption	0.01	0.06	0.27	0.51	0.74	0.94	0.93	0.80	0.61	0.40
Residential Investment	0.16	0.39	0.49	0.55	0.54	0.21	-0.05	-0.20	-0.25	-0.29
		Economy I: $\gamma = 0.82$ **and** $\pi = 0.2$								
GDP	0.02	-0.03	0.39	0.61	0.81	1.00	0.81	0.61	0.39	0.17
Consumption	0.01	0.01	0.10	0.29	0.49	0.88	0.73	0.68	0.60	0.55
Residential Investment	0.20	0.19	0.18	0.32	0.37	0.53	-0.12	-0.15	-0.21	-0.12
		Economy II: $\gamma = 0.0$ **and** $\pi = 0.8$								
GDP	0.02	0.17	0.39	0.61	0.81	1.00	0.81	0.61	0.39	0.17
Consumption	0.01	0.06	0.29	0.53	0.77	0.92	0.90	0.78	0.62	0.43
Residential Investment	0.10	0.35	0.52	0.69	0.78	0.84	0.69	0.51	0.31	0.09

This table displays the statistic properties of the real data and the simulated data. The first panel is the real data, and the other three panels are the simulated data sets, which are obtained by choosing different parameters in the model. Note that GDP in each model is given exogenously and consist with the real data.

Table 3: Granger Causality Test

	GDP	Consumption	Residential Investment
Data			
	GDP	Consumption	Residential Investment
GDP	/	N	Y
Consumption	N	/	Y
Residential Investment	N	N	/
Benchmark Economy: $\gamma = 0.82$ **and** $\pi = 0.8$			
	GDP	Consumption	Residential Investment
GDP	/	N	Y
Consumption	N	/	Y
Residential Investment	N	N	/
Economy I: $\gamma = 0.82$ **and** $\pi = 0.2$			
	GDP	Consumption	Residential Investment
GDP	/	N	N
Consumption	Y	/	N
Residential Investment	Y	Y	/
Economy II: $\gamma = 0.0$ **and** $\pi = 0.8$			
	GDP	Consumption	Residential Investment
GDP	/	Y	N
Consumption	N	/	N
Residential Investment	N	N	/

We test the Granger-Causality for the real data and the simulated data. Following Green (1997), we choose six lags for each variable in the regression and the significant level at 1%. 'Y' denotes that the variable in the corresponding column Granger-causes the variable in the corresponding row; and 'N' denotes that the variable in the corresponding column does NOT Granger-cause the variable in the corresponding row.

current purchases, including housing purchases, to intertemporally smooth his/her consumption. Because his/her current income does not increase, he/she must use his/her savings to finance the increased expenditures. He/she is able to borrow at a low rate of interest for most housing purchases, and this is not possible for purchases of other types of consumption. As a result, the agent will spend more on housing than on other goods. In other words, the accessibility of credit through mortgages makes residential investment respond more quickly to the signals of future TFP shocks. This explains why residential investment leads consumption and GDP. If the signal turns out to be accurate, the agent will achieve a higher income and become less financially constrained than he/she is today. At this time, he/she will be able to increase his/her consumption of other goods, which explains why consumption tends to coincide with GDP.

To highlight the mechanism at work in Economy I, it is also worthwhile to review the agents' policy functions at various wealth levels. We use the percentage difference between policy functions when the signals shift from bad to good $\Delta g_h(w)_{\{i,\xi,z,s_b,s_g\}}$ and $\Delta g_c(w)_{\{i,\xi,z,s_b,s_g\}}$ to attain this objective. Δg_h denotes the percentage difference in the policy of house purchases. Δg_h is defined by

$$\Delta g_h(w)_{\{i,\xi,z,s_b,s_g\}} = \frac{g_h(\bar{h}, w, i, \xi, z, s_g) - g_h(\bar{h}, w, i, \xi, z, s_b)}{g_h(\bar{h}, w, i, \xi, z, s_b)},$$

where $\bar{h}$ is chosen to be the mean of the house holdings in the economy; w denotes the individual wealth; and $s_b < z < s_g$ means that the signals shift from bad to good. Similarly, $\Delta g_c(w)_{\{i,\xi,z,s_b,s_g\}}$ denotes the percentage difference of the consumption policy when the signals shift from bad to good, and satisfies

$$\Delta g_c(w)_{\{i,\xi,z,s_b,s_g\}} = \frac{g_c(\bar{h}, w, i, \xi, z, s_g) - g_c(\bar{h}, w, \epsilon, z, s_b)}{g_c(\bar{h}, w, i, \xi, z, s_b)}.$$

Figure 2 depicts the two functional curves of $\Delta g_h(w)_{\{i,\xi,z,s_1,s_2\}}$ and $\Delta g_c(w)_{\{i,\xi,z,s_1,s_2\}}$ over w. In this figure, we choose (i, ξ, z) to be the middle values in their respective ranges.

Based on Figure 2, three observations can be made. First, the percentage adjustments in the areas of housing and consumption decrease with wealth. This indicates that the income effect of the good signals matters more for poor agents than it does for rich agents because the expected lifetime income of the poor increases more based on good signals. For more affluent individuals, liquid wealth, like housing and financial assets, constitutes the majority of their lifetime income. Hence, lifetime income does not change much based on the news shocks, and neither do purchases. Secondly, as can be seen from the right-hand side of the black dotted line, for rich people who are not limited by financial constraints, the percentage adjustment in consumption is roughly equal to the change in housing expenditure. This trend is consistent with the log utility function, which displays unity income elasticity. Finally, as seen on the left-hand side of the black dotted line, poor agents make much larger adjustments in housing capital than in consumption in the event of good news. This pattern is consistent with the idea that mortgages allow more rapid adjustments in housing purchases in response to information shocks than are possible for consumption. In summary, in the benchmark economy, poor people make the largest adjustments in response to good signals. These people are financially constrained and consequently adjust their residential investments more than they adjust their consumption. Because poor people constitute the majority of the population, adjustments to housing expenditure (residential investment) lead the economy.

In Economy I, which lacks news about the future TFP, the agents can only adjust consumption and residential investment simultaneously after observing the current aggregate shocks. Hence, both consumption and the residential investment reach their highest point at $T = 0$, which means that they both coincide with GDP. The Granger-Causality

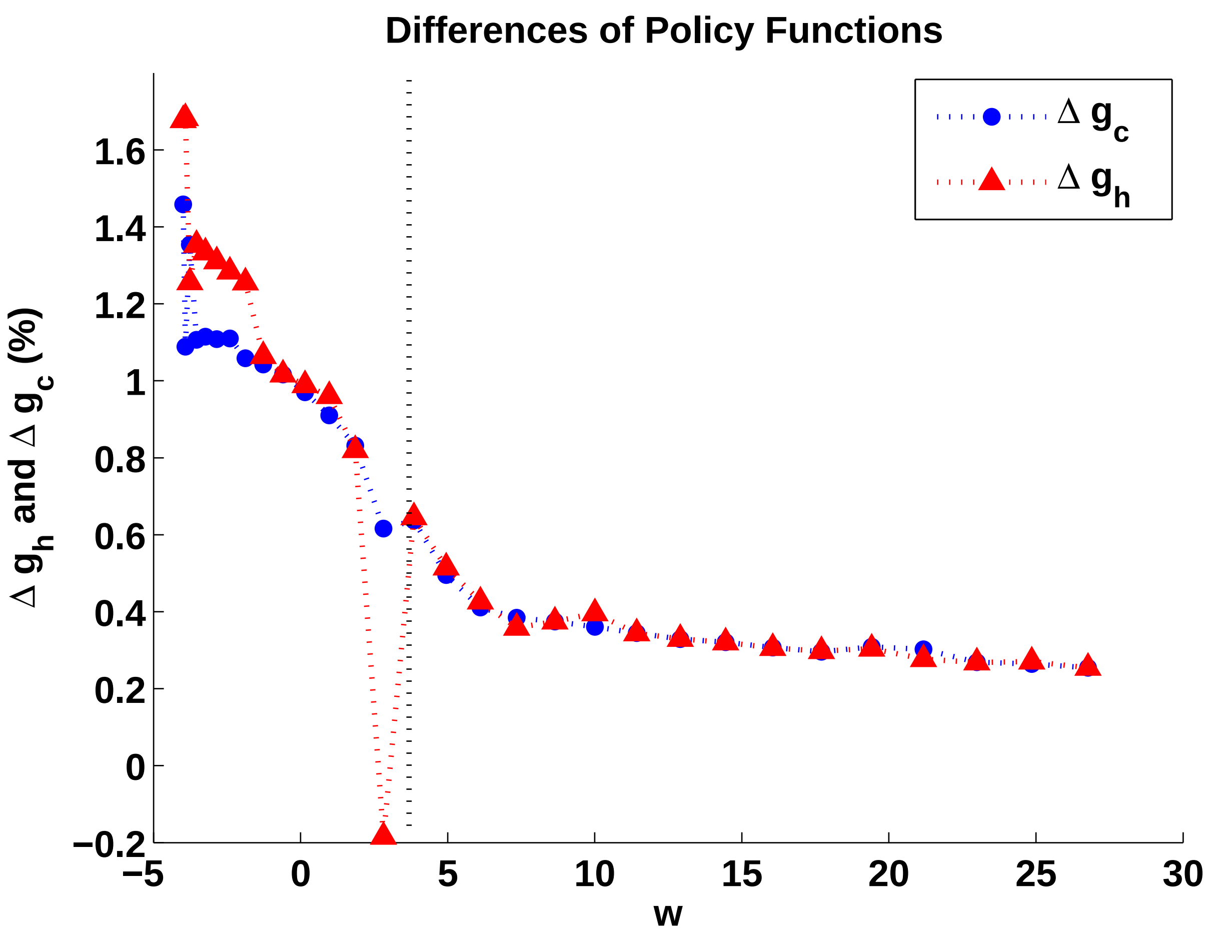

Figure 2: Δg_h vs. Δg_c

test also reveals that the residential investment does not Granger-cause consumption or the output. Because the agents can borrow against the value of their houses, they can adjust their house purchase levels more than is possible for consumption. Therefore, after $T = 0$, the correlation coefficients decline so much that they even reach a negative value. This indicates that the agents change their residential investment levels after observing current aggregate shocks, so much so that they can even make reverse adjustments later.

In Economy II, although the agents receive signals about future productivity, their financial constraints keep them from quickly responding to information shocks. Those who are experiencing financial constraints must wait to make adjustments until the next period, when they will have the higher income realized. Therefore, residential investment shares certain cyclical features with consumption. The highest correlation coefficients in both cases are with current GDP. Granger-Causality tests also confirm that residential investment does not Granger-cause consumption or output. It is interesting to compare this result with the one that we achieve in analyzing Economy I. In this economy, the lack of access to mortgages makes residential investment behave like consumption in its dynamic features; i.e., it acts as though agents do not receive any informative news.

Comparison of the numerical results of the three models leads us to conclude that only Benchmark Economy indicates that residential investment leads consumption, and that consumption coincides with GDP. This suggests that the two assumptions of news shocks and collateral constraints are crucial in generating the cyclical features of residential investment consistent with the U.S. data.

5 Conclusion

This paper establishes a dynamic stochastic partial equilibrium model that can be used to explain residential investment dynamics in the United States, focusing on the distinctive cyclical features of residential investment that it leads the entire economy. This paper departs from the existing literature in adding three new features to the model that help to generate the dynamics of residential investment in accordance with the data. These three distinctive assumptions are news shocks, collateral constraints and agent heterogeneity. The partial equilibrium analysis in which interest rates are exogenously fixed shows that the assumptions of news shocks and collateral constraints are essential to the generation of the conditions under which residential investment leads consumption and GDP. The key mechanism for these results is that in response to good information, agents purchase houses more than other goods because they are bound by collateral constraints.

We can definitely achieve more significant results by expanding this model from a partial equilibrium model to a general equilibrium model. By introducing the production sector into the model, we could explore how the economy shapes residential investment and business investment over time. In particular, business investment has a special feature in that it lags behind the whole economy. What does this fact reveal about agent choices regarding consumption, residential investment and business investment? Furthermore, how does this choice affect aggregate dynamics of the economy such like volatility and even social welfare? These questions should be addressed by future research.

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A Appendix

A.1 Data

This section describes the data with which the second moments are computed. The quarter data are from NIPA, including consumption, residential investment, output from 1959:I to 2008: IV. The consumption is composed of non-durable goods, durable goods and services. We subtract the imputed rents of self-owned houses from the consumption. The residential investment includes the costs of building new houses and renovating the old ones. The output consists of the consumption and the residential investment. Because the model we use is an endowment economy, we do not consider the business investment. Hence, we exclude the business investment from the subjects of research and similarly government expenditure. All the data are real and detrended with the Hodrick-Prescott filter.

A.2 Discretizing Two-Dimensional AR(1) Process

We extend the application of the method proposed by Tauchen (1986) from one-dimensional AR(1) process into the following two-dimensional AR(1) process defined by the following

$$z' = \rho z + \nu,$$

$$s = \rho z + \eta,$$

where z is the current productivity, s is the current signal and z' denotes the next-period productivity. The disturbances (ν, η) are i.i.d processes and satisfy the following conditions:

$$(\nu, \eta) \perp z$$
$$(\nu, \eta) \sim N(0, \Pi),$$

where Π is defined by:

$$\Pi = \sigma^2 \begin{bmatrix} 1 & \pi \\ \pi & 1 \end{bmatrix}.$$

Following Tauchen (1986), we assume the members of $\mathcal{Z}$ evenly-spaced and denoted as $\mathcal{Z} = \{z_1, z_2, \ldots, z_{n-1}, z_n\}$ which satisfies

$$z_1 < z_2 < \cdots < z_n.$$

The upper and the lower bounds on the range, z_1 and z_n, respectively, are set to m unconditional standard deviations on each side of 0.[5] Therefore, z_1 and z_n satisfy the following

$$z_1 = -m\frac{\sigma}{\sqrt{1-\rho^2}}$$

and

$$z_n = m\frac{\sigma}{\sqrt{1-\rho^2}},$$

We also assume the signals $s \in \mathcal{S} = \mathcal{Z}$. If we define the state composed of the current productivity and the signal, i.e. (z, s), the probability transition matrix is determined as follows.

Let $\omega = z_i - z_{i-1}$ $(i > 1)$. Given (z, s), the probability of z' satisfies the following.

[5] 0 is the unconditional mean of z in this paper.

If $1 < i < n$,

$$\begin{aligned}
&\text{Prob}(z' = z_i | z = z_m, s = z_h) = \\
&\text{Prob}(z_i - \omega/2 \leq \rho z_m + \nu \leq z_i + \omega/2 | \eta = z_h - \rho z_m) = \\
&\Phi\left[\frac{z_i + \omega/2 - \rho z_m - \rho(z_h - \rho z_m)}{\sigma\sqrt{(1-\rho^2)}}\right] - \Phi\left[\frac{z_i - \omega/2 - \rho z_m - \rho(z_h - \rho z_m)}{\sigma\sqrt{(1-\rho^2)}}\right]
\end{aligned}$$

If $i = 1$,

$$\begin{aligned}
&\text{Prob}(z' = z_i | z = z_m, s = z_h) = \\
&\text{Prob}(\rho z_m + \nu \leq z_1 + \omega/2 | \eta = z_h - \rho z_m) = \\
&\Phi\left[\frac{z_1 + \omega/2 - \rho z_m - \rho(z_h - \rho z_m)}{\sigma\sqrt{(1-\rho^2)}}\right]
\end{aligned}$$

If $i = n$,

$$\begin{aligned}
&\text{Prob}(z' = z_i | z = z_m, s = z_h) = \\
&\text{Prob}(\rho z_m + \nu \geq z_n - \omega/2 | \eta = z_h - \rho z_m) = \\
&1 - \Phi\left[\frac{z_n - \omega/2 - \rho z_m - \rho(z_h - \rho z_m)}{\sigma\sqrt{(1-\rho^2)}}\right]
\end{aligned}$$

where $\Phi(\cdot)$ denotes the cumulated distribution function of standard normal distribution.

Also, we know that given z', the probability of s' satisfies the following

If $1 < j < n$,

$$\begin{aligned}
&\mathrm{P}(s' = z_j | z' = z_i) = \\
&\mathrm{P}(z_j - \omega/2 \leq \rho z_i + \eta \leq z_j + \omega/2) = \\
&\Phi\left[\frac{z_j + \omega/2 - \rho z_i}{\sigma}\right] - \Phi\left[\frac{z_j - \omega/2 - \rho z_i}{\sigma}\right]
\end{aligned}$$

If $j = 1$,

$$\begin{aligned}
&\mathrm{P}(s' = z_1 | z' = z_i) = \\
&\mathrm{P}(\rho z_i + \eta \leq z_1 + \omega/2) = \\
&\Phi\left[\frac{z_1 + \omega/2 - \rho z_i}{\sigma}\right]
\end{aligned}$$

If $j = n$,

$$\begin{aligned}
&\mathrm{P}(z' = z_n | z' = z_i) = \\
&\mathrm{P}(\rho z_i + \nu \geq z_n - \omega/2) = \\
&1 - \Phi\left[\frac{z_n - \omega/2 - \rho z_i}{\sigma}\right]
\end{aligned}$$

Then the transition probability is defined by

$$\mathrm{Prob}(z', s' | z, s) = \mathrm{Prob}(z' | z, s)\mathrm{Prob}(s' | z').$$

A.3 Parameters of Income Process

This appendix displays the parameters of the processes of the individual income. The set of states for the individual income process (normalized to 1) is given by

$$\begin{pmatrix} 0.0352 & 0.1477 & 0.0563 & 0.2363 & 0.0900 & 0.3781 & 0.0564 \end{pmatrix}.$$

And the transition matrix Ω is given by

$$\begin{pmatrix} 0.8335 & 0.0084 & 0.1565 & 0.0016 & 0.0000 & 0.0000 & 0.0000 \\ 0.0084 & 0.8335 & 0.0016 & 0.1565 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.8335 & 0.0084 & 0.1565 & 0.0016 & 0.0000 \\ 0.0000 & 0.0000 & 0.0084 & 0.8335 & 0.0016 & 0.1565 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.8335 & 0.0084 & 0.1581 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0084 & 0.8335 & 0.1581 \\ 0.1581 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.8419 \end{pmatrix}.$$

The state of 1 corresponds to $n = 1$ of the age and the low state of idiosyncratic income. The state of 2 corresponds to $n = 1$ of the age and the high state of idiosyncratic income. And so on so forth until the state 7. The state 7 corresponds to the state of retirement.